VOLUME 18 SEPTEMBER, 1930 NUMBER 9

PROCEEDINGS
of
The Institute of Radio Engineers

I R E

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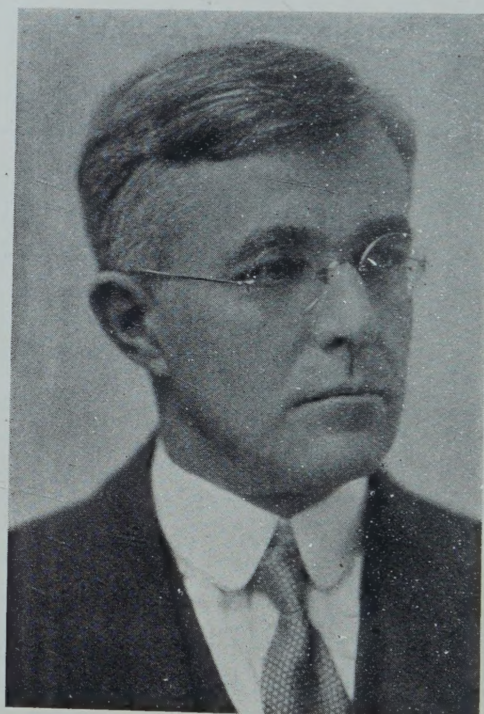
Institute of Radio Engineers
Forthcoming Meetings

DETROIT SECTION
September 19, 1930

NEW YORK MEETING
October 1, 1930

PITTSBURGH SECTION
September 16, 1930



Irving Langmuir
President of the Institute, 1923

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Irving Langmuir, twelfth president of the Institute, was born in Brooklyn, N. Y., January 31, 1881. After being graduated from Columbia University in 1903 with the degree of Metallurgical Engineer, he spent three years in post graduate work in Germany at the University of Göttingen under Professor Nernst, where he received his Ph.D. degree.

Upon his return to America in 1906 he became an instructor in chemistry at the Stevens Institute of Technology entering the research laboratory of the General Electric Company at Schenectady three years later. He is at present associate director of the research laboratory.

Dr. Langmuir's researches have been conducted in the fields of chemistry, physics, and engineering. His most notable contribution to the field of science has been his pioneer work in electronics. In December, 1913, he published the results of his studies on the effect of space charge and residual gases upon thermionic currents in high vacuum. This investigation was the beginning of a long series of experiments and inventions made during the following years which have led to the development of the modern high vacuum power engineering. His development of the mercury condensation pump and his studies of the characteristics of tungsten and thoriated tungsten filaments have contributed greatly to the success of vacuum tube projection.

Among the best known of Dr. Langmuir's inventions is the gas-filled incandescent tungsten lamp. In 1925 he developed the atomic hydrogen welding process which is opening up new possibilities in electric welding.

Dr. Langmuir has been the recipient of distinguished honors in the field of chemistry, physics, and engineering and has been awarded the Nichols medal given by the New York Section of the American Chemical Society in 1915, and again in 1920; the Hughes medal, awarded by the Royal Society of London in 1918; the Rumford medal of the American Academy of Arts and Sciences in 1921; the Cannizzara prize, awarded by the Royal National Academy of the Lincei, Rome, Italy, in 1925; the Perkin medal of the American Section of the Society of Chemical Industry, 1928; and the School of Mines medal, Columbia University in 1929.

He has received honorary degrees from the University of Edinburg, Northwestern University, Union College, Columbia University, Kenyon College, and the Technische Hochschule, of Berlin-Charlottenburg.

He is an Honorary member of the Royal Institution of Great Britain and the Deutsche Gesellschaft für Technische Physik; a Fellow of the American Physical Society and of the Institute of Radio Engineers; a Member of the National Academy of Sciences, the American Academy of Arts and Sciences, and the American Chemical Society.

He became an Associate member of the Institute of Radio Engineers in 1914 and a Fellow in 1922.

INSTITUTE NEWS AND RADIO NOTES

Cosmic Data Broadcasts

We are indebted to Science Service, Washington, D. C. for the following information concerning broadcast of Cosmic Data.

Coöperating with the American Section of the International Scientific Radio Union, Science Service will collect daily data on terrestrial magnetism, the solar constant, sun spots, and the auroral displays in order that they may be broadcast and otherwise distributed to those interested.

The U. S. Navy by broadcasting the daily cosmic data message, the U. S. Army by transmitting by radio to Washington data from outlying points, the U. S. Coast and Geodetic Survey by furnishing magnetic data, the Mount Wilson Observatory and the U. S. Naval Observatory by furnishing sun spot data, the Astrophysical Observatory of the Smithsonian Institution by furnishing solar constant values, participate actively and fundamentally in the project.

Broadcast Schedule

The cosmic data broadcasts were inaugurated on August 1, 1930. They are transmitted daily, including Sunday, from the Navy radio station, NAA, Washington, as an addition to the weather message directed at French radio station, FYL, Lafayette, at time 16:00 zone plus 5 (4 p.m. Eastern Standard Time) frequency 16,060 kilocycles

Code Used

The letters URSI is the distinguishing sign at the beginning of the cosmic data message. URSI are the initials of the Union Radio Scientifique Internationale (International Scientific Radio Union). Each class of data is coded separately and preceded by an identifying word: SOL for solar constant, MAG for terrestrial magnetism, SUN for sun spots, AURO for auroras. The data are expressed in a number code in groups of five, similar to that used in the transmission of meteorological information. Plain English will be added when extraordinary phenomena demand it. The message is signed SCIENSERVC, the cable address of Science Service.

Sol (Solar Constant)

First figure indicates day of week

1 Sunday

2 Monday

3 Tuesday
4 Wednesday
5 Thursday
6 Friday
7 Saturday

Second, third and fourth figures:

Decimal fractional portion of solar constant value; add one to obtain complete value. When 933 is transmitted, the solar constant is 1,933 calories. The solar constant of radiation is defined as the total intensity of solar radiation outside the earth's atmosphere at the earth's mean distance from the sun and is expressed in calories per square centimeter per minute; the calorie, being the amount of heat required to warm one gram of water at 15 deg. C through one deg. C; the unit intensity of radiant intensity being defined as that which, if completely absorbed by a surface at right angles to the beam, will produce one calorie of heat per square centimeter per minute.

Fifth figures indicate whether the solar constant value is considered to be:

3 Satisfactory
5 Satisfactory minus (not quite satisfactory)
7 Unsatisfactory

Example: SOL 79333, Saturday, solar constant, 1,933 calories, satisfactory. Solar constant data are supplied by the Astrophysical Observatory of the Smithsonian Institution and are the averaged values of the solar constant determinations of that organization's observers at Montezuma station, Chile.

MAG (TERRESTRIAL MAGNETISM)

First Group

First figure in first group shows day of week:

1 Sunday
2 Monday
3 Tuesday
4 Wednesday
5 Thursday
6 Friday
7 Saturday

Second figure of first group indicates character of day:

3 Quiet day
5 Day of moderate disturbance
7 Greatly disturbed day

Third figure of first group indicates:
- 3 Day marked by bay, a disturbance lasting only an hour or so, with departure from normal curve in one direction only
- 5 Day marked by rapid pulsations
- 7 Day marked by long period pulsations or oscillations
- 9 Day marked by irregular oscillations
- X Not used

Fourth figure of first group indicates that second group gives time of:
- 3 Beginning of disturbance
- 5 End of disturbance
- 7 Beginning of disturbance; end given in third group
- X Not used

Fifth figure is unused and is sent as X

Second Group

Gives Greenwich mean time of beginning or ending of disturbance as indicated by the fourth figure of the first group. (If there is a beginning and end on the same day, a third group will give time of ending.)

First and second figures: Hours, preceded by zero if less than ten.

Third and fourth figures: Minutes, preceded by zero if less than ten.

Fifth figures: Tenths of minutes, in the case of a sudden commencement of a disturbance. Other times will be given to whole minutes only and X will be fifth figure.

Third Group

See explanation above.

Example: MAG 1535X 08407, Sunday, day of moderate disturbance marked by bay. Disturbance ends 08:40.7 G. M. T.

Terrestrial magnetic data are supplied by the U. S. Coast and Geodetic Survey from its observatory at Tucson, Arizona. Period covered is for 24 hours preceding 14 hours G. M. T. of the Greenwich and local day of the week as given by the first figure of the first group.

SUN (SUN SPOTS)

First figure indicates day of week:
- 1 Sunday
- 2 Monday
- 3 Tuesday
- 4 Wednesday

5 Thursday
6 Friday
7 Saturday

Second and third figures indicate number of groups of sun spots, preceded by zero if less than ten.

Fourth and fifth figures indicate total number of sun spots, preceded by zero if less than ten.

Example: SUN 10314, Sunday, three groups of sun spots containing a total of fourteen spots.

Wolf Number equals X (10 g plus s) where g is the number of groups, s the number of spots and K a constant, for Mt. Wilson about 0.77. The Wolf Number of the example given above is about 34.

Plain English descriptions of unusual solar phenomena will be added where necessary.

Sun spot data are furnished by the Mount Wilson Observatory of the Carnegie Institution of Washington, Pasadena, California, from observations made at about 16 hours G. M. T. (8 A.M. Pacific Standard Time) or as soon thereafter as weather permits.

Auro (Auroras)

Code for transmission of data on auroras will be formulated later as observations in Alaska will not begin until fall of 1930.

Example of Cosmic Data Message

URSI SOL 79333 MAG 1535X 08407 SUN 10314 SCIENSERVC

Other Distribution

Upon request, Science Service will transmit the cosmic data message telegraphically over commercial channels, tolls collect. If desired, the numerals will be rendered into the following syllable code to reduce tolls:

Code—(1	2	3	4	5	6	7	8	9	0	X
(ba	de	fi	go	ki	am	en	ip	ot	ux	vy

Example above would be sent as:

URSI SOL ENOTFIFIFI MAG BAKUFIKUVY UXIPGOUXEN SUN BAUXFIBAGO SCIENSERVC

Science Service will compile weekly in mimeograph form the data of the daily cosmic data messages and upon specific request will distribute them by mail to those who can utilize or distribute the information further. The scientific magazine, *Terrestrial Magnetism*, published by Johns Hopkins Press (Editor: Jno. A. Fleming, acting director,

Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.) will publish summaries of the cosmic data.

Science Service will also utilize the information of the cosmic data messages in the preparation of its service to newspapers in such a way that the public will be kept informed of the occurrence of notable changes in the phenomena reported and the possible effects upon earthly conditions.

Those interested in correlating the cosmic data with other phenomena and in studying the literature upon the fields affected by the cosmic data reported will be placed in communication with competent authorities upon application to Science Service.

Further Cosmic Data

If demand arises and as information becomes available, it is proposed to add to the cosmic data message information such as follows:

Terrestrial Electricity—Predominant direction of natural earth-currents registered for the preceding 24 hours. At present no earth-current system is installed in this country, but one is planned.

Radio Phenomena—Signal intensity for long and short wave reception at representative stations has been suggested for inclusion in the message.

Solar Activity—In addition to sun spots, it has been suggested there might be included values for the daily intensity of other phenomena on the sun's surface such as character figures for bright hydrogen flocculi, calcium flocculi, dark hydrogen flocculi, magnetic character-number prominences, faculae, etc.

Within the next year the Carnegie Institution's Magnetic Observatory at Huancayo, Peru, expects to broadcast magnetic and earth current data which may be available in the cosmic data message. A radio station is being installed at the Watheroo, Australia, station of the Carnegie Institution and within a year Washington expects to be in direct radio communication with that station. Data from that station may then be available for inclusion in the cosmic data message. It is hoped that those to whom the cosmic data messages are useful will report how the data are used and suggest how the service can be improved.

French Cosmic Data Broadcasts

Since December 1, 1928, the French Government has been radiating a cosmic data message from the Eiffel Tower station, and later at the request of Americans interested, these messages were later repeated from Lafayette and Issy-les-Moulineaux so that the following schedule is now in effect:

Station	Location	Frequency Kilocycles	Wavelength Meters	Time
FLE	Eiffel Tower, Paris	207.5	1445	11:20 G.M.T.
FYL	Lafayette, Bordeaux	15.9	18900	20:30 G.M.T.
	Issy-les-Moulineaux, near Paris	922.5	32.5	20:30 G.M.T.

The messages are in plain French language and not in code. They include information on the following:

(a) Steadiness or disturbance of the earth's magnetic field.

(b) Steadiness or disturbance of atmospheric electric field.

(c) Apparent activity of the solar surface, as regards both sun spots and faculae.

These data are supplied by Physical Institute of the Globe of Paris, the National Meterological Office, the Astronomical Observatory at Meudon, near Paris, and the Val Joyeuz Observatory, near Versailles.

The French cosmic data broadcasts are being copied and used by some American radio observers. The Navy communications office is copying them daily when reception conditions make it possible, and the American Radio Relay League at the request of A. E. Kennelly is planning tests to determine how far inland the short wave signals of Issy-les-Moulineaux can be copied.

Origin of URSI Cosmic Data Broadcast

When American radio engineers were informed of the French cosmic data broadcasts, A. E. Kennelly of Harvard as a representative of the committee on coöperation of the American Section of the International Radio Union (URSI) distributed the information to those interested. This led to an informal meeting of scientists at the National Academy of Sciences, Washington, D. C., in April, 1929, at which it was decided that a daily cosmic data message to be broadcast from an American government radio station would be desirable.

This led to the appointment of a special joint committee of the URSI and American Geophysical Union on the subject, which outlined the project and asked the coöperation of Science Service in the collection of the daily cosmic data messages. Those coöperating formally or informally in the formative stages of the project included: Dr. A. E. Kennelly of Harvard, chairman coöperation committee, American Section, URSI; Dr. Charles F. Marvin, chief of the U. S. Weather Bureau and chairman, American Geophysical Union Committee on Cosmic Broadcasts; Commander N. H. Heck of the U. S. Coast and Geodetic Survey; John A. Fleming, acting director, and Dr. M. A. Tuve, of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington; Major W. R. Blair and Lieut W. T. Guest,

Signal Corps, U. S. Army; Capt. S. C. Hooper and Lt. Commander J. R. Redman, U. S. Navy; Dr. J. H. Dellinger, of the Bureau of Standards, technical secretary of the American Section, URSI; Dr. L. W. Austin, chairman of the American Section, URSI; Dr. Seth B. Nicholson of Mount Wilson Observatory; Dr. W. J. Humphreys and E. B. Calvert of the U. S. Weather Bureau; Dr. A. Hoyt Taylor and Dr. L. P. Wheeler of the Naval Research Laboratory, Bellevue, Anacostia, D. C.

Proceedings Binders

Because of the enlarged size of the PROCEEDINGS published during 1929, many of our members find that they are unable to fit the twelve issues into the standard binder which has been available in the past.

We are pleased to announce that a larger size of binder is now available which will hold the twelve issues published during 1929.

When ordering the larger size be sure to specify that the large binder is desired. They are available at $1.75 each and the member's name will be stamped on it for 50 cents additional. The smaller size binder is still available at $1.50.

Associate Application Form

For the benefit of members who desire to have available each month an application form for Associate membership, there is printed in the PROCEEDINGS a condensed Associate form. In this issue this application will be found on page XXXIII of the advertising section.

Application forms for the Member or Fellow grades may be obtained upon application to the Institute office.

The Committee on Membership asks that members of the Institute bring the aims and activities of the Institute to the attention of desirable and eligible nonmembers. The condensed form in the advertising section of the PROCEEDINGS each month may be helpful.

Radio Signal Transmissions of Standard Frequency July to December, 1930

The following is a schedule of radio signals of standard frequencies for use by the public in calibrating frequency standards and trans-

mitting and receiving apparatus as transmitted from station WWV of the Bureau of Standards, Washington, D. C.

Further information regarding these schedules and how to utilize the transmissions can be found on pages 10 and 11 of the January, 1930, issue of the PROCEEDINGS, and in the Bureau of Standards Letter Circular No. 171, which may be obtained by applying to the Bureau of Standards, Washington, D. C.

Eastern Standard Time	Sept. 22	Oct. 20	Nov. 20	Dec. 22
10:00 P.M.	500	1600	4000	550
10:12	600	1800	4400	600
10:24	700	2000	4800	700
10:36	800	2400	5200	800
10:48	1000	2800	5800	1000
11:00	1200	3200	6400	1200
11:12	1400	3600	7000	1400
11:24	1500	4000	7600	1500

INSTITUTE MEETINGS

CHICAGO SECTION

The July 11th meeting of the Chicago Section held in the Engineering Building, was called to order by H. E. Kranz, chairman of the Section.

The report of the Nominating Committee was read and the following officers for the coming year were elected unanimously: chairman, Byron B. Minnium; vice-chairman, Samuel E. Adair; secretary-treasurer, J. Barton Hoag. In addition, Harvey G. Hayes was elected as a member of the Executive Committee.

Following the election of officers, H. E. Kranz, chief engineer of the Grigsby-Grunow Company, commented upon several reels of film showing manufacturing processes employed in the plant of the Grigsby-Grunow Company.

DETROIT SECTION

At the June 20th meeting of the Detroit Section, held in the Detroit News Building, A. B. Buchanan, chairman of the Section presiding, two papers were presented.

The first paper of the evening on "Television" by L. N. Holland of the electrical engineering department of the University of Michigan, considered the subject under six divisions as follows: scanning of the image, converting light impulses into electrical impulses, transmitting, reconverting the electrical impulses into light impulses, spreading the light impulses over a screen, synchronization.

The difficulties involved in the satisfactory solution of each part of the problem were discussed. The talk was closed with a description of E. F. W. Alexanderson's latest achievement, in the projection of television images for theatre use. Special attention was paid to the light valve based upon the invention of A. Karolus.

The second paper of the evening on "New System of Power Filtration" was delivered by S. M. Hanley of the Ray Vox Engineering Company of Detroit.

The papers were discussed by Messrs. Buchanan, Case, Firestone, and others.

At the election of officers for the coming year, the following were selected: chairman, L. N. Holland; vice chairman, S. L. Bailey; secretary-treasurer, Samuel Firestone.

Rochester Section

On May 15th a joint meeting of the Rochester Sections of the American Institute of Electrical Engineers, the Institute of Radio Engineers, and the Rochester Engineering Society was held at the Eastman School of Music. Earl Karker, Chairman of the I.R.E. Section, presided.

F. L. Hunt of the Bell Telephone Laboratories, delivered an illustrated paper on "Recording and Reproducing Sound Pictures." In addition to lantern slides, several talking pictures, illustrating the various problems and apparatus involved, were shown.

Prior to the joint meeting, a business session of the Rochester Section was held at which the following new officers were elected: chairman, H. E. Gordon; vice-chairman, I. G. Maloff; secretary-treasurer, H. A. Brown.

The annual report of the secretary and treasurer was presented and accepted.

The attendance at the meeting totaled four hundred and eighty-one.

Personal Mention

There appeared in the July issue an incorrect statement to the effect that H. B. Closson had left the Atwater-Kent Manufacturing Company to become affiliated with the RCA-Victor Company at Camden, N. J. The correct name is L. E. Closson who is an engineer for the RCA-Victor Company. H. B. Closson, Jr. continues with the Atwater-Kent Manufacturing Company.

Lieutenant Mervin W. Arps is now located at the Naval Research Laboratory at Washington, D. C.

Previously with the Radiomarine Corporation of America at Tuckerton, N. J., Preston de Grauw Baldwin is now service engineer for RCA Photophone, Inc.

John Bargamian has left the Plymouth Electric Company to become a service engineer for the Canadian Westinghouse Company.

Charles S. Breeding, formerly general manager of the Aero Radio Corporation in Los Angeles, is now in the engineering department of the Western Air Express at Alhambra, California.

Carl H. Butman, formerly secretary to the Federal Radio Commission has returned to his radio consulting practice, acting also as advertising counsel.

Previously with the Mackay Radio and Telegraph Company, Edward N. Dingley has joined the engineering department of RCA Radiotron Company at Harrison, N. J.

Clifford J. Dow has left the Federal Telegraph Company at Palo Alto, California, to enter the service of Heintz and Kaufman of San Francisco as a radio engineer.

W. Robert Ferris has become research engineer for the RCA Radiotron Company at Harrison, N. J. having left the General Electric Company at Schenectady, N. Y.

H. H. Friend has left the National Broadcasting Company to enter the engineering department of RCA Communications.

W. E. Garity is now chief engineer for the Disney Film Recording Company at Hollywood having previously been an operation engineer for De Forest Phonofilm, Inc.

John B. Hawkins is now factory manager for the Cordonic Manufacturing Corporation of Holland, Michigan, having previously been with the United Reproducers Corporation.

L. E. Hayslett formerly with the United Reproducers Corporation has entered the radio engineering department of the General Motors Radio Corporation at Dayton.

P. M. Honnell has joined the technical staff of the Bell Telephone Laboratories.

W. C. Little has entered the radio test department of the General Electric Company at Schenectady.

Frederick S. Mockford, previously in charge of wireless at Croydon Airport is now with the Marconi Wireless Telegraph Company in London.

Raymond W. Newby, formerly with the American Insurance Union is now chief engineer of WABC of the Columbia Broadcasting System.

C. A. Petry has left the Westinghouse Electric and Manufacturing Company at Chicopee Falls to enter the radio engineering department of the Bell Telephone Laboratories.

Pinckney B. Reed has left the Stewart-Warner Corporation to become an installation engineer for RCA Photophone, Inc.

F. H. Schnell, formerly connected with Aero Products, Inc., has become chief of staff of the Radio and Television Institute of Chicago.

O. B. Gunby has left the engineering department of the East Pittsburgh plant of the Westinghouse Electric and Manufacturing Company to join the RCA-Victor Company at Camden, N. J.

Previously section engineer with the Westinghouse Electric and Manufacturing Company at Chicopee Falls, Mass., John B. Coleman is now a radio engineer for the RCA-Victor Company.

Theodore A. Smith, formerly in the engineering department of the Radio Corporation of America, is now with the RCA Victor Company at their New York City office.

Albert E. Snow, previously with the Wireless Specialty Apparatus Company is now in the employ of the Radiomarine Company at Chatham, Mass.

Daniel W. Wells has left the Springfield, Mass., branch of the Westinghouse Electric and Manufacturing Company to join the staff of the RCA-Victor Company.

Joseph T. Wissmann, previously with the Radio Corporation of America is now recording engineer for the Vitaphone Corporation of Brooklyn.

H. D. Hineline has left the International Telephone and Telegraph Company to join the patent department of United Research Corporation at Long Island City.

Errata

Fig. 7 in the paper on "A Tuned-Reed Course Indicator for the Four- and Twelve-Course Aircraft Radio Range" by F. W. Dunmore which appeared on page 971 of the June, 1930, issue of the PROCEEDINGS should be omitted and Fig. 8 substituted for it. A new Fig. 8 is given below. The cut labels as given in the original paper are correct for these revised figures.

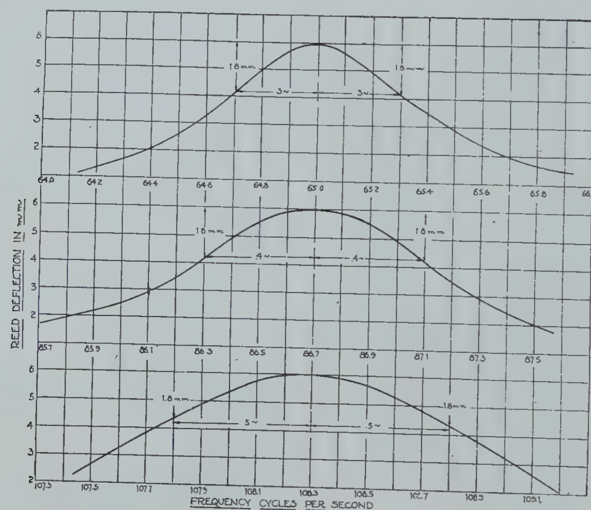

FIG. 8

PART II

TECHNICAL PAPERS

Proceedings of the Institute of Radio Engineers
Volume 18, Number 9 *September, 1930*

LONG WAVE RADIO RECEIVING MEASUREMENTS AT THE BUREAU OF STANDARDS IN 1929*

By

L. W. Austin

(Laboratory for Special Radio Transmission Research, Washington, D. C.)

Summary—*This paper gives monthly averages of daylight signal intensity at Washington for 1929, as received from a number of European and American low-frequency stations.*

Several of the stations formerly measured have ceased to transmit regularly at hours suitable for all-daylight transmission paths and their measurement has therefore been discontinued. The annual average field intensities of the European stations have not shown much change since last year, but the atmospheric disturbances have increased.

THE MONTHLY averages of field intensity of various long wave stations and their corresponding atmospheric disturbances as measured in 1929, and the yearly averages 1923–1929, are given in the following tables and curves.

The signals marked A.M. were measured between 10 and 11 A.M., E. S. T., and represent all-daylight transmission paths except in the case of Nauen, Germany, during the short days of winter. The P.M. signals measured between 3 and 4 P.M., E. S. T., have a transmission path partly in daylight and partly in darkness, except for a few days in midsummer when Rugby and Carnarvon have all-daylight paths.

On account of their irregularity of transmission, Cayey, Porto Rico, and San Diego, California, have not been included this year in the tables. Ste. Assise (FTU) has not generally been transmitting during the time of our morning observations, while Bolinas has not been heard since January, 1929. Caracas, Venezuela (AYB) (f=40 kc, λ= 7500 meters) has been added to the list of stations measured. This station is fairly regular in transmitting and affords an opportunity for the continuation of the study of the south-north transmission paths, interrupted by the lack of signals from Monte Grande and Cayey.

Table I (page 1485) gives the approximate transmission data for the stations measured.

Fig. 1 shows the changes of the morning yearly averages of some of the stations which have been measured since 1923. Bordeaux (FYL) shows the same intensity as in 1928, while the shorter wave Ste. Assise station (FTT) has fallen considerably from its 1928

* Dewey decimal classification: R113.2 Original manuscript received by the Institute, June 27, 1930. Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

value.[1] The average intensity of Nauen (DFW) is slightly higher in 1929 than in 1928.

The observations in Fig. 1 which represent all-daylight trans-

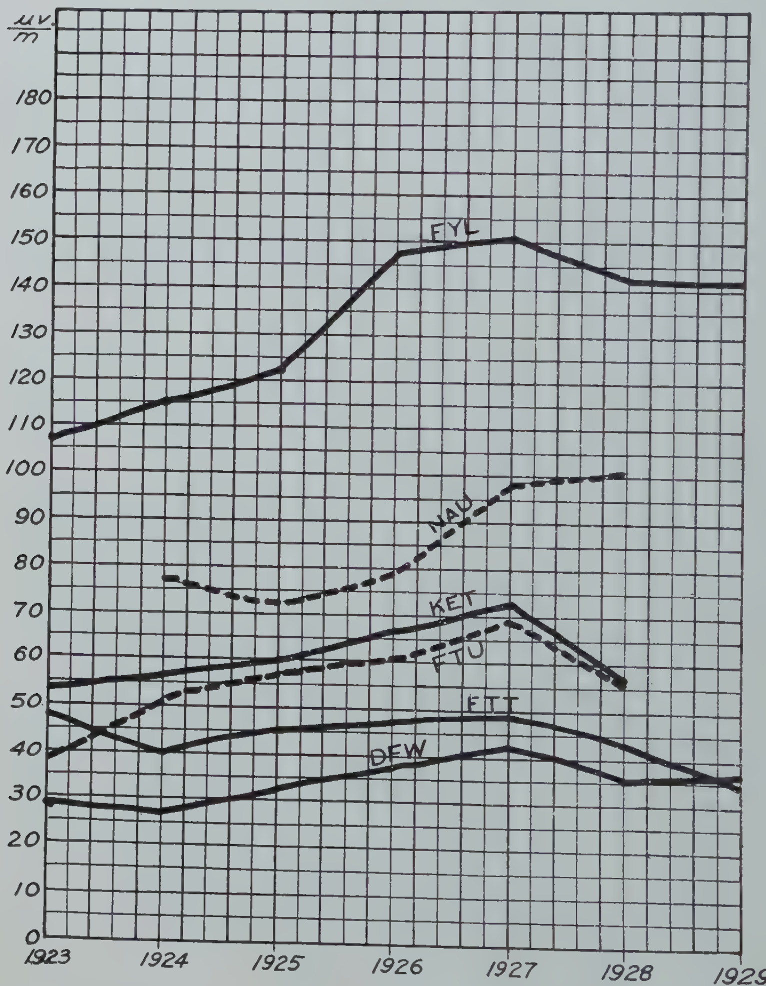

Fig. 1

mission do not show the uniform drop in intensity in 1929 which might have been expected from the decrease in the number of sun spots.

The annual averages of afternoon signal intensities and atmospheric disturbances are shown in Fig. 2. It will be noticed that Bordeaux has fallen slightly since 1928, while Ste. Assise (FTU) has risen con-

[1] The average for (FTT) in 1929 is for seven months only.

siderably. The shorter wave Ste. Assise station (FTT) and Nauen (DFW) have not changed appreciably. The atmospheric disturbances on both wavelengths shown are considerably stronger than in 1928.

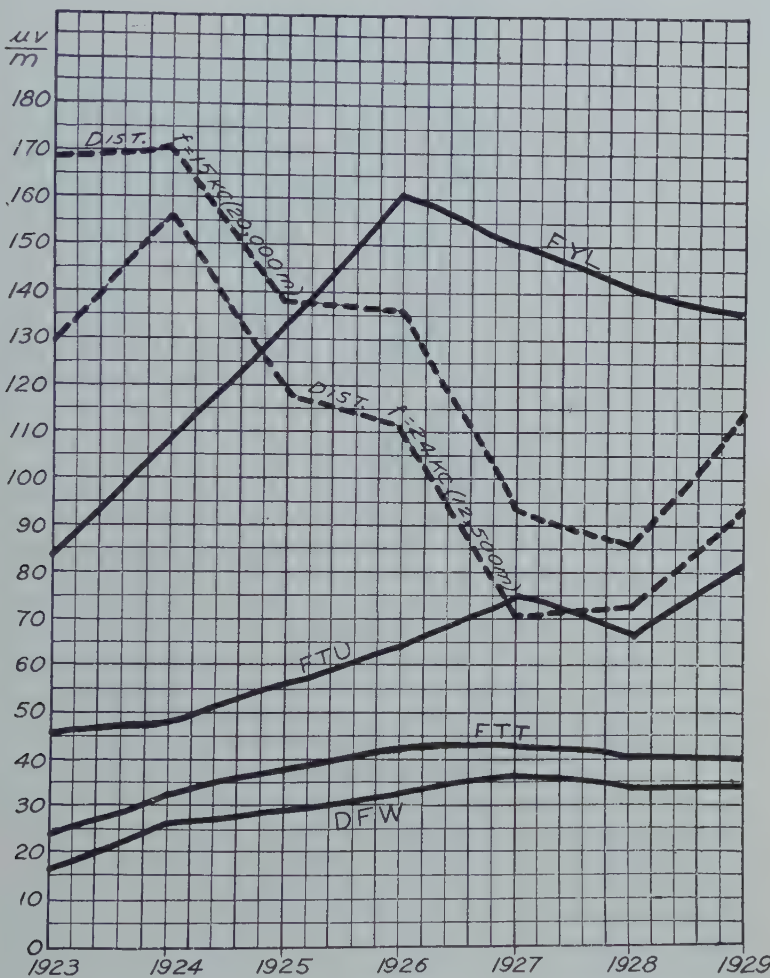

Fig. 2

Fig. 3 shows the monthly distribution of atmospherics in 1927, 1928, and 1929 at a wavelength of 20,000 meters (15 kc).

Table V (page 1486) gives the monthly averages of the signal strength of some stations of the Radio Corporation of America at New Brunswick and Tuckerton, New Jersey; Rocky Point, Long Island; and Marion, Massachusetts, as received at Washington. There

is a marked decrease in the strength of both New Brunswick stations during the last four months of 1929. The reported antenna current would not explain this signal drop. The other Radio Corporation stations mentioned showed slight decreases in signal at the close of the

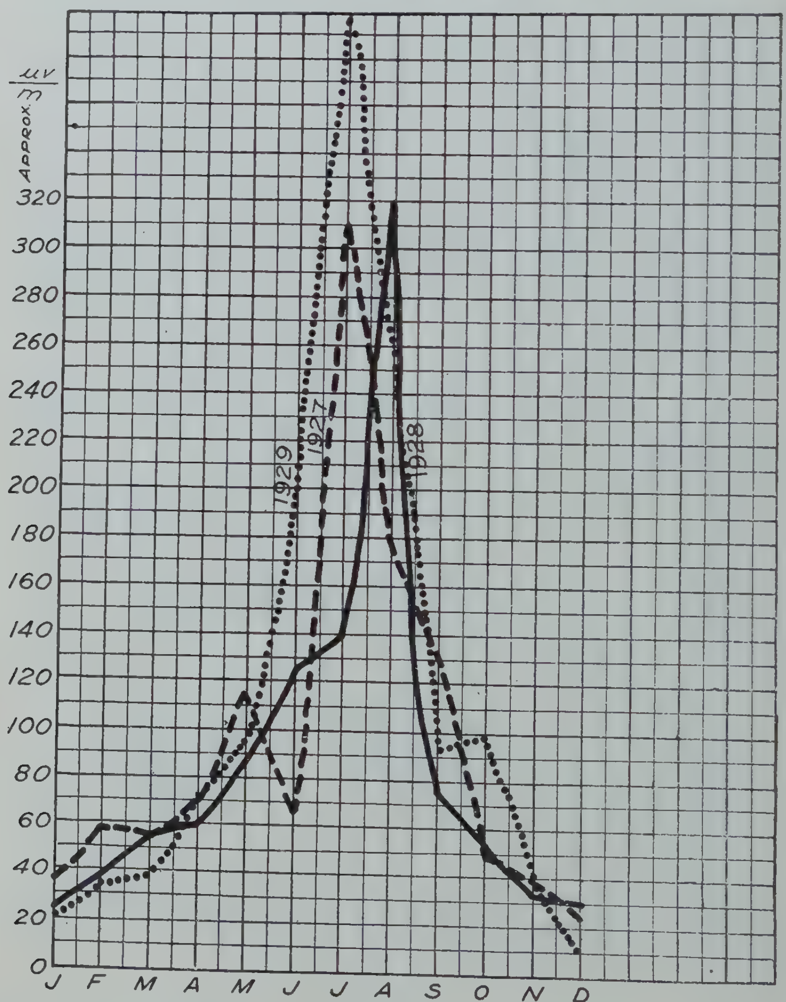

Fig. 3

year. The monthly values of the Rocky Point stations (WSS) and (WQK) are incomplete as the two stations transmitted alternately in periods of two weeks.

The yearly averages of the signal intensities of the Radio Corporation stations were slightly lower in 1929 than in 1928.

TABLE I

TRANSMISSION DATA, 1929

		Frequency f(kc)	Wave-length, (m)	Approximate Antenna Current I(amperes)	Approximate Effective Height h(meters)	Distance from Washington d(km)
FYL	Bordeaux, France.	15.9	18,900	500	180	6160
FTT	Ste. Assise, France	21.0	14,250	350	180	6200
FTU	Ste. Assise, France	15.2	19,710	475	180	6200
DFY	Nauen, Germany	16.6	18,060	367	170	6650
DFW	Nauen, Germany	23.0	13,000	379	130	6650
GBR	Rugby, England	16.0	18,740	708*	185	5930
PCG	Kootwijk, Holland	16.8	17,800	325	185	6100
IRB	Rome, Italy	20.8	14,400	156	500	7160
GLC	Carnarvon, Wales	31.7	9,450	67	300	5840
NAU	Cayey, Porto Rico	33.8	8,870	100	120	2490
NPL	San Diego, California	30.0	10,000	90	120	3700
WCI	Tuckerton, N. J.	18.4	16,304	893	96	251
WGG	Tuckerton, N. J.	22.1	13,575	690	57	251
WII	New Brunswick, N. J.	21.8	13,750	650	68	281
WRT	New Brunswick, N. J.	22.6	13,265	630	68	281
WSO	Marion, Mass.	25.8	11,620	500	66	660
WSS	Rocky Point, L. I., N. Y.	18.8	15,957	700	83	435
WQK	Rocky Point, L. I., N. Y.	18.2	16,465	650	83	435
KET	Bolinas, California	22.9	13,100	600	51	3920
AYB	Caracas, Venezuela	40.0	7,500	—	—	3200

* Six months average.

TABLE II

AVERAGE SIGNAL INTENSITY AND ATMOSPHERIC DISTURBANCES FOR BORDEAUX (FYL), RUGBY (GBR), STE. ASSISE (FTU), NAUEN (DFY), AND KOOTWIJK (PCG), IN MICROVOLTS PER METER

	A.M.					P.M.					
1929	FYL	GBR	DFY	PCG	Dist.	FYL	GBR	FTU	DFY	PCG	Dist.
Jan.	142	155	59	59	23	213	232	93	88	82	22
Feb.	142	186	67	54	25	240	260	94	83	73	36
Mar.	156	—	71	64	30	221	234	89	76	69	38
Apr.	140	154	56	51	40	111	122	67	47	41	71
May	135	153	60	58	42	71*	69*	38*	31*	31*	95
June	136	132	59	57	44	70*	85*	—	31*	30*	200
July	155	—	64	60	44	77*	—	—	41*	41*	397
Aug.	140	150	63	62	46	81*	—	—	35*	35*	260
Sept.	144	108	51	54	21	73	46	—	26	31	93
Oct.	108	82	40	54	62	94	67	—	38	51	98
Nov.	142	113	48	50	34	178	156	—	63	68	41
Dec.	130	117	60	49	16	197	183	104	81	80	17
Av.	142	135	58	56	36	136	145	81	53	53	114

* Uncertain owing to heavy atmospheric disturbances.

TABLE III

AVERAGE SIGNAL INTENSITY AND ATMOSPHERIC DISTURBANCES FOR ROME (IRB), STE. ASSISE (FTT), NAUEN (DFW), IN MICROVOLTS PER METER

	A.M.				P.M.			
1929	IRB	FTT	DFW	Dist.	IRB	FTT	DFW	Dist.
Jan.	41	—	32	18	72	65	57	20
Feb.	53	—	31	20	67	60	49	29
Mar.	59	—	47	24	71	63	53	31
Apr.	48	—	37	32	39	34	25	58
May	49	38	36	35	25*	22*	17*	84
June	49	35	40	38	24*	—	20*	174
July	58	—	43	36	28*	—	20*	336
Aug.	48	47	39	37	27*	—	24*	215
Sept.	59	48	39	15	27	25	9	71
Oct.	46	30	30	47	37	28	23	73
Nov.	32	18	27	19	41	21	41	23
Dec.	48	32	39	12	68	39	53	12
Av.	49	35	37	28	44	40	33	94

* Uncertain owing to heavy atmospheric disturbances.

TABLE IV

AVERAGE SIGNAL INTENSITY AND ATMOSPHERIC DISTURBANCES FOR CARNARVON (GLC) AND CARACAS (AYB), IN MICROVOLTS PER METER.

	A.M.			P.M.	
1929	GLC	AYB	Dist.	GLC	Dist.
Jan.	16	—	18	25	—
Feb.	18	—	20	23	—
Mar.	34	—	24	28	—
Apr.	19	—	32	—	—
May	15	33	35	—	84
June	18	31	38	—	174
July	17	41	27	—	230
Aug.	14	39	25	—	142
Sept.	14	—	15	—	51
Oct.	22	14	65	17	86
Nov.	13	4	19	17	23
Dec.	18	7	13	19	13
Av.	18	24	28	20	100

TABLE V

AVERAGE SIGNAL INTENSITY FOR NEW BRUNSWICK, N. J. (WII AND WRT), TUCKERTON, N. J. (WCI AND WGG), ROCKY POINT, L. I. (WQK AND WSS), AND MARION, MASS. (WSO), IN MICROVOLTS PER METER

	A.M.							P.M.						
1929	WII	WRT	WGG	WCI	WQK	WSS	WSO	WII	WRT	WGG	WCI	WQK	WSS	WSO
Jan.	2.8	3.2	3.5	4.1	1.5	2.5	1.2	3.0	3.1	3.4	4.1	—	2.4	1.1
Feb.	2.9	3.1	3.2	3.8	1.9	2.5	1.3	3.1	3.3	3.5	4.0	1.9	2.6	1.3
Mar.	2.6	2.5	2.7	3.4	1.7	2.3	0.9	2.9	3.0	3.0	3.7	1.8	2.5	1.1
Apr.	2.5	2.7	2.8	3.4	—	2.4	1.0	2.5	2.5	2.6	3.4	—	2.2	0.8
May	2.4	2.4	2.6	3.0	1.4	—	0.9	2.4	2.4	2.5	2.9	1.3	1.8	0.7
June	2.6	2.4	2.8	2.9	1.5	1.7	1.0	—	2.3	2.7	2.9	1.2	1.5	0.6
July	2.6	2.4	2.5	3.0	1.5	1.9	0.9	—	2.3	—	2.9	1.3	1.7	0.5
Aug.	2.1	2.3	—	3.0	1.7	1.9	0.8	—	2.3	—	2.8	1.3	1.7	0.6
Sept.	1.0	1.2	—	2.8	—	1.5	0.5	0.9	0.9	—	2.1	—	1.3	0.5
Oct.	1.0	1.0	—	2.8	1.1	1.3	0.5	0.9	1.0	—	2.2	0.9	0.9	0.4
Nov.	0.9	1.2	1.9	3.1	1.5	1.6	0.6	1.2	1.3	1.8	3.3	1.3	1.7	0.7
Dec.	1.4	2.1	2.6	3.7	2.0	2.3	0.9	1.5	2.0	2.7	3.9	2.1	2.5	1.1
Av.	2.1	2.2	2.7	3.3	1.6	2.0	0.9	2.0	2.2	2.8	3.2	1.5	1.9	0.8

The continuous automatic recording of the field intensities of the Radio Corporation stations has been continued and there is now available a very large number of 24-hour records for investigation.

The constantly decreasing use of long wave transmission in radio communication is causing anxiety as to the continuity of this work in the future.

Mimeographed copies of the daily observations of signal intensities and of the strength of atmospherics are available for distribution to those interested.

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LOW-FREQUENCY RADIO TRANSMISSION*

By

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Summary—*This paper presents the results of field intensity measurements on low-frequency transmission (17.8 kc) from the R.C.A. station WCI, located a, Tuckerton, N. J. The results of observations made at Newton Centre and at Medfordt Mass., are presented. Apparatus for alternating antenna observations with variously oriented loop observations is described and the results of such measurements are presented. The relation of the state of elliptical polarization of received signals to loop responses for various positions of the loop is investigated theoretically, and the results thus obtained are applied to an interpretation of the observations. The necessary approximations required to render this problem determinate are pointed out and their questionable validity emphasized.*

The mean of the received signal intensity in the absence of magnetic disturbances is lowest during the night, and strong sunrise and sunset peaks are found. An inversion of received signal intensity is noted during magnetic storm conditions. At such times the night field strength exceeds the day field.

DURING the last few years observations of radio transmission phenomena have been undertaken by numerous observers, and have served greatly to enhance our understanding of the phenomena. While this mass of data has furnished a firm basis for the Kennelly-Heaviside layer theory of radio transmission, it has disclosed phenomena so erratic and complex that a satisfactory and reasonably complete quantitative or even qualitative theory seems almost as much in the future as ever.

The observations contributed have by no means been confined to the time-honored measurements of received field strengths on particular frequencies and over particular transmission paths, but include ingeniously devised transmissions designed to disclose the characteristics of the transmitting medium. Such experiments in the last few years have played an increasingly important part in our study of the mechanism of transmission.

This paper, however, will be confined to a consideration of the possibilities (and the limitations) of observations of the most straightforward and time-honored type; i.e., those of field intensities. It is hoped that this discussion may serve to emphasize some of the difficulties and advantages peculiar to such observations.

* Dewey decimal classification: R113. Original manuscript received by the Institute, June 21, 1930. Presented before Fifth Annual Convention of the Institute, August 19, 1930.

I. Possible Correlations with Transmission

Numerous investigators have demonstrated quantitatively that striking correlations exist between measured fields and solar and magnetic elements.[1,2,3]

Provided sufficiently long transmission paths are chosen, the correlations observed are found to be fairly uniform over similar but somewhat varied transmission paths and over narrow ranges of the frequency spectrum. Observations in widely different frequency regions or on the same transmission over widely different transmission paths (as for instance East-West vs North-South) disclose quite

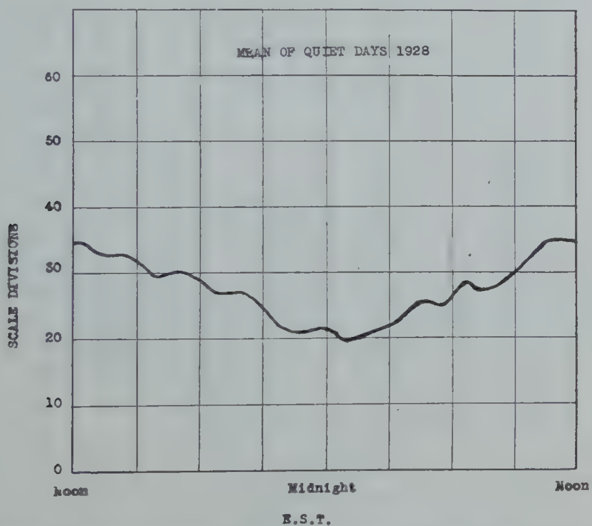

Fig. 1—Mean WCI reception at Newton Centre with moderately quiet magnetic elements. (Mean from June to December, 1928.)

different but distinct correlations. Conditions are much more complicated, however, if observations over shorter transmission paths are attempted. In this case relatively slight differences in transmission paths or frequency may alter profoundly the observed results (at least inasmuch as the graph of received signal intensity is concerned).

The situation described above is, of course, quite understandable from the point of view of the Kennelly-Heaviside layer transmission theory, for in the case of the shorter distance transmission a very limited number of important paths may supply nearly equal fields at

[1] Pickard, G. W., Proc. I. R. E., 15, 63–97; February, 1927; 749–766; September, 1927.
[2] Austin, L. W., Proc. I. R. E., 15, 825–843; October, 1927.
[3] Anderson, L. N., Proc. I. R. E., 16, 297–347; March, 1928.

the point of observation, and important phase interference and polarization phenomena may be manifested which may, moreover, be greatly altered by slight changes of path or frequency. At longer distances a multiplication of the number of paths and (supposedly) an increase in the importance of attenuation render these phase phenomena of relatively less importance (at least in low-frequency transmission) and more consistent results are hence obtainable.

Phase interference and polarization phenomena are, however, probably very important in most of the results reported, as the data themselves frequently emphasize.[4]

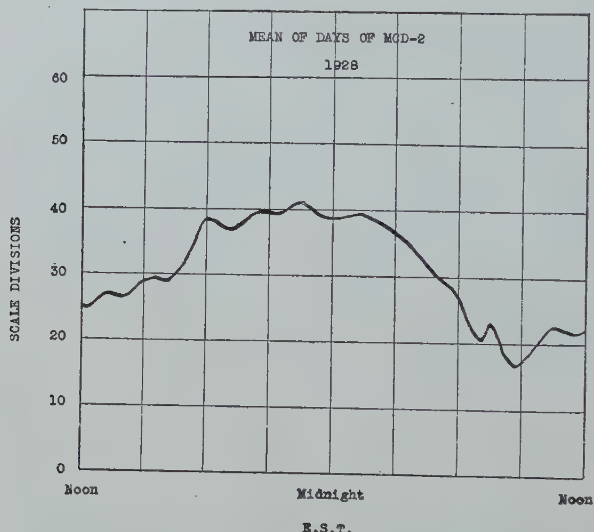

Fig. 2—Mean WCI reception at Newton Centre during magnetic storm conditions.

II. Characteristics of Low-Frequency Reception over Short Transmission Paths

Low-frequency reception over short transmission paths furnishes striking evidence of phase interference phenomena and of polarization. At such low frequencies, the received intensity on an antenna over moderate distances is, moreover, in general a fairly consistent function of time with a distinct diurnal periodicity. The mean of 24-hour reception curves for WCI at Newton Centre, Mass., is shown in Fig. 1, which shows the mean over an observation period of six months (exclusive of periods of marked magnetic disturbances). The mean

[4] Potter, R. K., Proc. I. R. E., **18**, 581–648; April, 1930.

of reception during the disturbed periods is shown in Fig. 2. Certain striking features should be noted.

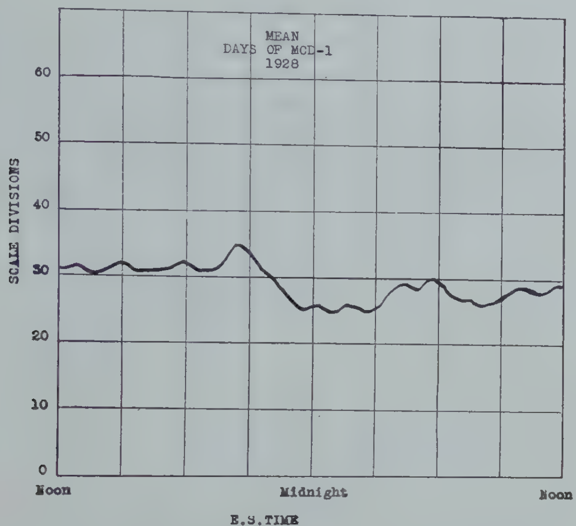

Fig. 3—Partial inversion due to minor magnetic disturbances.

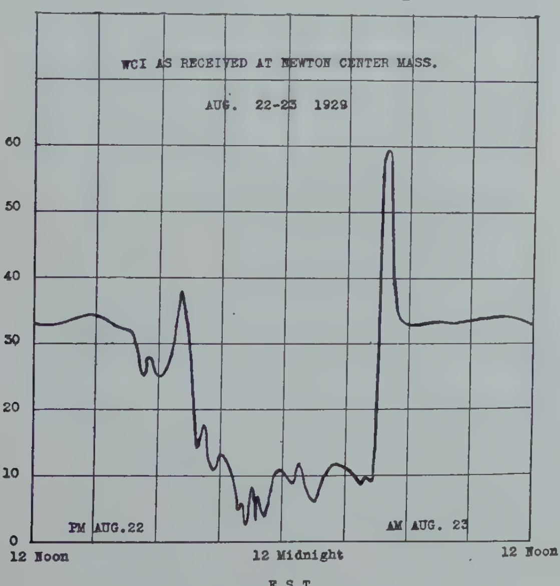

Fig. 4—Typical magnetically quiet reception showing marked sunset and sunrise peaks and low night fields.

(1) The normal curve shows a high daytime level, marked sunrise and sunset peaks, and low signal intensity during the night.

(2) The magnetically-disturbed observations show low daytime values and high night values.

These effects are much more evident when single records are chosen, for they are, of course, smoothed out considerably by the long time means, with consequent variable times for the sunrise and sunset peaks, variable times of occurrence of maximum to minimum field-strength ratio, etc. The striking ratios obtainable on individual low-frequency records over short transmission paths such as Tuckerton-Boston are shown in Figs. 4 and 5, where individual records, corresponding to

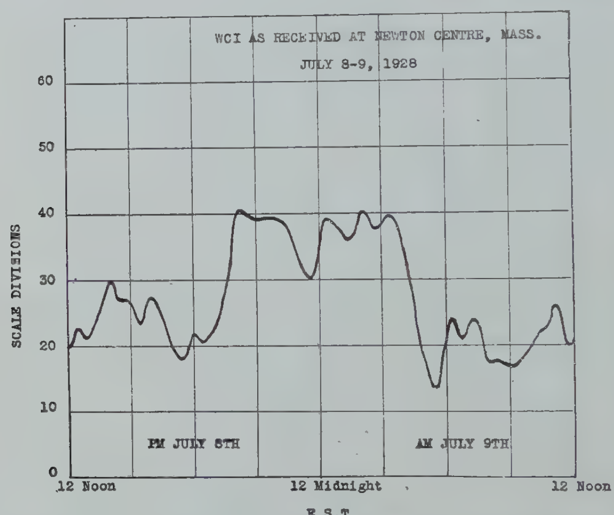

Fig. 5—Reception during the severe magnetic storm of July, 1928, showing the marked inversion.

magnetically quiet and magnetically disturbed days, are plotted. The inversion during magnetic storms is a function of the severity of the storm, and the results for 1928 are hence most striking because of the extremely severe storm during the month of July, 1928. A similar tendency to inversion, however, is consistently noted during periods of lesser disturbances. This is shown distinctly by the flattening tendency for days of magnetic character, as Fig. 3 illustrates.

The graphical patterns obtainable at other points of observation, or on other frequencies and over other transmission paths are by no means always identical with those indicated. Some indications of these relationships is to be found in recently published papers in the PROCEEDINGS[5,6] reporting observations on the same station at other

[5] Austin, L. W., PROC. I. R. E., **17**, 1192–1205; July, 1929.
[6] Kenrick and Jen, PROC. I. R. E., **17**, 2034–2052; November, 1929.

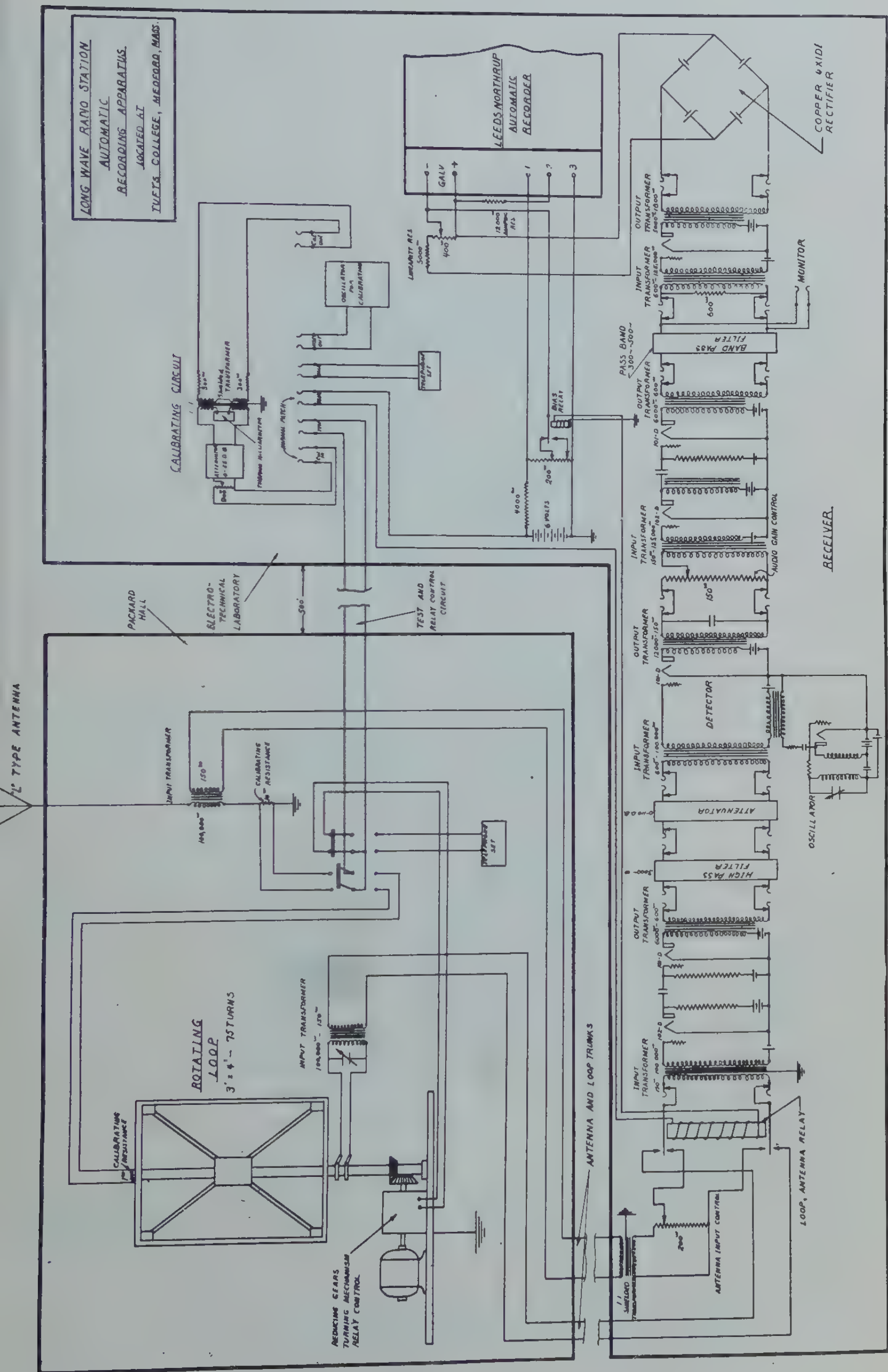

Fig. 6—Schematic diagram of recorder circuit.

points, and quasi-consistent cyclic variations in the patterns are sometimes noted, but such observations are not available in sufficient numbers to render positive quantitative conclusions in regard to these phenomena justified at present.

III. Interpretation of Observed Results

The observed phenomena reported above seem fairly consistent with the accepted Kennelly-Heaviside layer theory, and were attributed to phase-interference phenomena and to rotations of the plane of polarization of the signal in transmission through the layer (in accordance with the Nichols and Schelleng theory).

In order to establish more definitely and quantitatively the factors contributing to the large changes of signal intensity observable on the set-up at Newton Centre employing reception on an antenna, a set-up was designed and installed at Tufts College at Medford, Mass., which would allow simultaneous (or nearly simultaneous) observations to be made of WCI signal intensity as observed on an antenna and on a loop variously oriented with respect to the transmitting station. Observations are now being carried on employing this equipment in place of that located at Newton Centre. A diagram of the recording apparatus is shown in Fig. 6.

In addition to recording the strength of signals as received by an antenna, a rotating loop was utilized. This loop does not rotate with a uniform velocity, but remains fixed in position for a period of five minutes, and is then swung into a new position in an interval of a few seconds. As it is at present operated, the loop is rotated through an angle of 45 deg. every five minutes, which gives four significant positions for each half revolution. A relay control, operated by the loop driving mechanism, controls two relays, one of which shifts the receiver input to the loop or antenna, while the other, operated simultaneously, controls the bias on the recorder and shifts the zero, so that the antenna and loop observations may be recorded without confusion by one recorder. This arrangement gives $2\frac{1}{2}$ minutes recording of antenna signal strength, followed by $2\frac{1}{2}$ minutes of loop signal strength in one of its positions; for example, the plane of the loop toward the station. This sequence of events is carried on continuously.

Peculiar local conditions made it desirable to locate both loop and antenna at a considerable distance from the receiver and recording mechanism. On account of the relatively low frequency of the station being recorded, no difficulty is experienced in trunking the antenna and loop signals over a distance of approximately 500 ft.

Fig. 6 shows the essentials of the recording apparatus. The re-

ceiving equipment appears rather elaborate in design, but it is to be remembered that for results to be of value the receiver must eliminate all interference, must be highly selective because of the close spacing of channels in this frequency range, and must be constant in gain, linear in response, and flexible for testing purposes. The physical appearance of the assembly is shown in Fig. 7.

Fig. 7. Front view of recorder amplifier.

Selectivity is obtained primarily by the use of a band-pass filter of only 200 cycles band width, located between the last two low-frequency amplifiers. This also serves to eliminate to a satisfactory degree both static and local interference.

Constant gain of the amplifier is obtained by using high-grade tubes, maintained at constant potentials. Oscillator stability is important, and is obtained by using a type of circuit which maintains

constant frequency to within a few cycles for reasonable ranges of temperature and tube potentials.

The incoming signal is stepped down to a frequency of 400 cycles, which is the mean frequency of the band-pass filter, and permits approximately 75 cycles variation in either direction without changing the amplification of the set more than 5 per cent.

Rectification is obtained by means of a copper oxide rectifier, which experience has shown to be very stable and practically linear in the circuit shown.

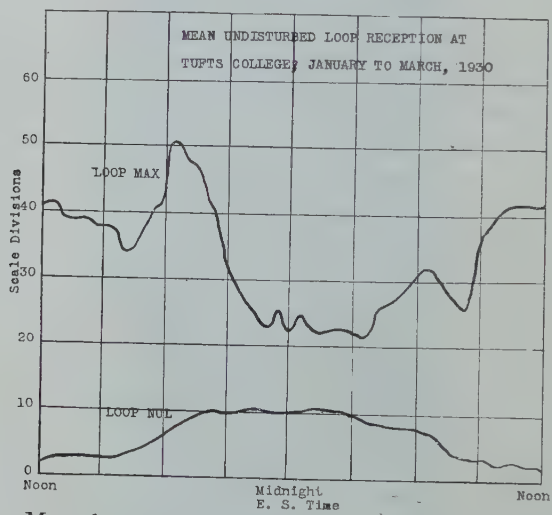

Fig. 9—Mean loop reception. Note polarization during night.

The recorder circuit is shown in detail; this equipment is checked periodically and the calibrator circuit is also shown. A typical actual record obtained by the use of this recorder is shown in Fig. 8.

It will be noted that the daytime values of field as observed on the loop constitute a normal lemniscate diagram, indicating a normal vertical polarization of the electric vector in the received signal. At the sunrise and sunset periods, however, corresponding to periods of marked change in field, (and frequently during the entire night period) the "nul" bearings disappear, and the intensity of the received signal frequently becomes greater in this position than in the maximum. These phenomena are well exhibited by the mean curves shown in Fig. 9, while a not infrequent case is shown in Fig. 10, where the "nul" reading exceeds the reading in the daytime maximum position.

In order to interpret the above phenomena in terms of the angles of polarization in the received electric vector, it is necessary to review

the relation between the state of polarization of an electromagnetic wave and the response in antennas oriented at various angles to the direction of propagation. This theory has already been developed, but it will be briefly derived and discussed here.

Review of Loop Reception Theory.[7]

In examining the response of a loop to an electromagnetic wave, we have merely to note that the resulting induced e.m.f. is directly proportional to the time rate of change of magnetic flux thru the loop and hence proportional to the resultant H of the wave normal to the

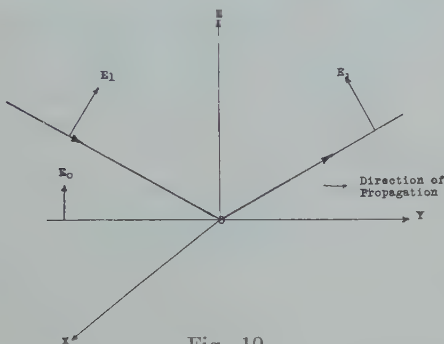

Fig. 10.

loop area (since the loop area is in general constant and the time variation of H sinusoidal).

In the following analysis we shall investigate the response of a loop oriented so that its area is

(1) normal to the magnetic vector of the ground wave (which we shall term the loop maximum position),

(2) parallel to the magnetic vector of the ground wave (which we term the loop "nul" position),

(3) at 45 deg. with these positions (2 such positions termed 45-deg. lead and 45-deg. lag positions).

We shall consider a wave composed of a normally polarized "ground wave" component and a single elliptically polarized downcoming wave.

Let us consider a wave system propogating along the y axis as shown in Fig. 10.

We assume normally polarized ground wave components E_0 and H_0 and an elliptically polarized downcoming wave in which we let

E_1 and H_1 = normally polarized electric and magnetic vectors of the downcoming wave,

[7] Appleton and Ratcliffe, *Proc. Royal Soc.*, **115**, 291–317, 1927.

E_0 and H_0 = electric and magnetic vectors in the ground wave (normally polarized, i.e., vertical electric vector),

E_2 and H_2 = electric and magnetic, vector perpendicular to E_1 and H_1 (abnormally polarized components),

$\mathcal{O}_1$ = phase angle of the electric vector of the normally polarized downcoming wave with respect to the electric vector of the ground wave,

$\mathcal{O}_2$ = phase angle of the electric vector of the abnormally polarized downcoming wave with respect to the electric vector of the ground wave,

α = angle of incidence of the downcoming wave, $\omega = 2\pi f$.

Maxwell's equations applied to the case shown in Fig. 8 give

$$E_x = 0 \qquad H_x = H_0 \sin \omega t + 2H_1 \sin (\omega t + \mathcal{O}_1)$$
$$E_y = 0 \qquad H_y = 2H_2 \cos \alpha \sin (\omega t + \mathcal{O}_2)$$
$$E_z = E_0 \cos \omega t + 2E_1 \sin \alpha \cos (\omega t + \mathcal{O}_1) \qquad H_z = 0 \tag{1}$$

For the loop in the "max" position, the response is proportional to dH_x/dt. Now if we note that the magnitudes of the H components are (except for trivial constants) equal to the corresponding E components i.e., $|H_1| \backsim |E_1|$, $|H_0| \backsim |E_0|$ etc., and further that like components due to the various waves must be added with due regard to differences in time phase we have for the voltage in the loop (utilizing equations 1)

$$E^2_{\text{max}} \sim E_0{}^2 + 4E_1{}^2 + 4E_0E_1 \cos \mathcal{O}_1 \tag{2}$$

Likewise, for the loop in the "nul" position, the response is proportional to dH_y/dt which in turn is proportional to

$$E^2_{\text{nul}} \sim 4(E_2)^2 \cos^2 \alpha \tag{3}$$

For the loop in the 45-deg. positions the response is an appropriate combination of (2) and (3), i.e.,

$$E^2_{\text{45deg.}} \sim \tfrac{1}{2}[(E_0 + 2E_1 \cos \mathcal{O}_1 \pm 2E_2 \cos \alpha \cos \mathcal{O}_2)^2 + (2E_1 \sin \mathcal{O} \pm 2E_2 \cos \alpha \sin \mathcal{O}_2)^2] \tag{4}$$

We may also write for the antenna reception (proportional to E_z)

$$E^2_{\text{ant}} \sim E_0{}^2 + 4E_1{}^2 \sin^2 \alpha + 4E_0E_1 \sin \alpha \cos \mathcal{O} \tag{5}$$

Interpretation of Results Using Loop Theory.

Equations (2) to (5) give us a number of useful equations with which to interpret antenna and loop observations such as those described above. Unfortunately, however, the number of variables involved are so large that a complete quantitative determination can only be made on the basis of somewhat precarious assumptions. The authors prefer not to engage at the present time in computations necessitating the use of such assumptions, but a brief outline of

possible approximations and of their validity and consequences is perhaps not without interest.

First of all it is important to note that, provided the ground wave remains constant (certainly this is far less reproachable than most of the necessary assumptions) and if the subscript t_1 denotes intensity at the time t_1 and the subscript t_2 the intensity at the time t_2 then for the antenna

$$(E^2_{\mathrm{ant}})_{t_1} = E_0{}^2 + 4(E_1{}^2)_{t_1}\,(\sin^2\alpha) + 4E_0(E_1)_{t_1}\,(\sin\alpha)_{t_1}\,(\cos\mathcal{O}_1)_{t_1} \tag{5}$$

and likewise

$$(E^2_{\mathrm{ant}})_{t_2} = E_0{}^2 + 4(E_1{}^2)_{t_2}\,(\sin^2\alpha)_{t_2} + 4E_0(E_1)_{t_2}\,(\sin\alpha)_{t_2}\,(\cos\mathcal{O}_1)_{t_2} \tag{6}$$

Now if we let $\delta(E^2_{\mathrm{ant}}) = (E^2_{\mathrm{ant}})_{t_1} - (E^2_{\mathrm{ant}})_{t_2}$ and if we further assume that t_1 and t_2 are chosen sufficiently near together and at such times that E_1 and $\sin\alpha$ vary negligibly compared with rapidly varying $\cos\mathcal{O}_1$ we have

$$\delta(E^2_{\mathrm{ant}}) = 4E_0E_1\sin\alpha\left[(\cos\mathcal{O}_1)_{t_1} - (\cos\mathcal{O}_1)_{t_2}\right] \tag{7}$$

A similar argument applied to (4) gives

$$\delta(E^2_{\mathrm{loop}}) = 4E_0E_1((\cos\mathcal{O}_1)_{t_1} - (\cos\mathcal{O}_1)_{t_2}) \tag{8}$$

Dividing (7) by (8) gives

$$\frac{\delta(E^2_{\mathrm{ant}})}{\delta(E^2_{\mathrm{loop}})} = \sin\alpha. \tag{9}$$

The difficulty with this method lies, however, in the necessary assumption that E_1 and α remain constant during periods of rapid change, investigations on high frequencies of layer heights and selective path fading indicates these assumptions may be indeed questionable (particularly at sunrise and sunset). Thus, it will be noted that rotations, as well as changes in attenuation may cause rapid changes in E_1 while layer-height changes will effect $\sin\alpha$ as well as $\mathcal{O}_1$ although the change in $\mathcal{O}_1$ would in general (probably) be the more important. In Appleton's experiments he was able to vary $\mathcal{O}_1$ rapidly by introducing small changes in transmitted frequency, but computations of the necessary frequency changes in the very low frequency region (i.e., near 18 kc) are not promising due to the large frequency changes necessary and the consequent serious interference involved to say nothing of apparatus difficulties. Pulser methods also do not appear promising since the length of the pulses necessary to produce suitable resolution is short compared with the time of one cycle of normal frequency.

Approximations for Small Sky Waves.

For $E_1 \ll E_0$ and $E_2 \ll E_0$ it is possible (to an approximate equivalent

to neglecting higher order terms in E_1/E_0 and E_2/E_0) to neglect the time quadrature components in (2) to (5) and write approximately

$$E_{\text{ant}} = E_0 + 2E_1 \sin\alpha \cos \Theta_1 \tag{10}$$

$$E_{\max} = E_0 + 2E_1 \cos \Theta_1 \tag{11}$$

$$E_{\text{nul}} = 2E_2 \cos\alpha \tag{12}$$

$$E_{45\text{deg.}} = \frac{1}{\sqrt{2}} (E_0 + 2E_1 \cos \Theta_1 \pm 2E_2 \cos\alpha \cos \Theta_2) \tag{13}$$

It is evident that under these conditions, if a method of increments similar to that already described is employed, it is possible to solve for $\sin\alpha$ on a similar set of assumptions as to the variability of $\sin\alpha$ and E_1, i.e.,

$$\frac{\Delta E_{\text{ant}}}{\Delta E_{\max}} = \sin\alpha \tag{14}$$

Furthermore, if the ground wave is known, the values of $E_1 \cos \Theta_1$, E_2 and $\cos \Theta_2$, as well as α may be computed.

However, in long-wave observations, it is by no means safe to assume that sky waves are small compared to the ground wave, and, inasmuch as even day values change by factors of 2 or 3 to 1 between normal and disturbed days, sky waves of considerable magnitude seem to exist in the daytime and no method of finding the true value of the ground wave is hence apparent.

Summary of Theory.

From the above analysis we see that while it is possible to deduce the electric and magnetic vectors in the downcoming wave from low-frequency field strength data computations along the lines indicated, the assumptions required to render the problem determinant are so questionable as to discourage attempts at such an interpretation at the present state of the observations, particularly as the transmission path is too long to render determinations of α accurate. It is preferred, therefore, to point out that the maximum and nul loop intensity curves show the values of

$$\sqrt{E_0^2 + 4E_1^2 + 4E_0E_1 \cos \Theta_1}$$

and $2E_2 \cos\alpha$ without attempting to segregate further these factors. In any case, however, we may conclude that the low antenna values observable (together with the much higher loop nul values observable at the same time) show that in the normal night field E_1 is at least comparable to one-half E_0 and that E_2 may be at least as great and probably frequently much greater than E_1. It remains for further observations (probably of still other types) to furnish sufficient

data to render the problem completely determinant, particularly when it is recalled that probably a number of reflections are of importance in determining the resultant field. It will be noted, however, that strong evidence is furnished by the loop observations for large rotations of the plane of polarization in the downcoming wave during the night (and particularly at sunrise and sunset) and the evidence also indicates a much more moderate change in amplitude of the resultant electric vector than might be inferred from antenna observations alone.

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CERTAIN FACTORS AFFECTING THE GAIN OF DIRECTIVE ANTENNAS*

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Summary—*This paper analyzes the performance of antenna arrays as influenced by certain variables within the control of the designing engineer. It starts with an extremely simple analysis of the interfering effects produced by two sources of waves of the same amplitude. This is followed by a short discussion of a paper by Ronald Foster, which considers two antennas and also 16 antennas when arranged in linear array. Two antennas separated in space by ¼ wavelength and in phase by ¼ period give sensibly more radiation in one direction than in the opposite. This, for convenience, has been called a unidirectional couplet. A number of these couplets may be arranged in linear array, thereby giving an extremely useful directive system. Diagrams are shown for such arrays as affected by the number and spacings of the individual couplets. The gains from such arrays are calculated and data are given showing fair agreement between calculation and observation.*

Directional diagrams for arrays of coaxial antennas indicate that somewhat less gain may be expected from this form than when the elements are spaced laterally. Combinations of these two types of arrays give marked directional properties in both their horizontal and vertical planes of reference. This principle has been used rather generally in short-wave communication. This paper also discusses effects resulting from combining two or more arrays. In one case the space between two arrays tends to emphasize spurious lobes. The directional diagram of such a combination may be rotated within limits by changing the phasing between adjacent arrays or sections of an array. In all of the above cases the influence of the earth is ignored.

A mathematical appendix gives general equations for calculating directional diagrams of linear arrays. Special cases of these equations apply to the figures included in the main part of the text. General equations are also given for calculating the gains of arrays. Similar equations permit the areas of diagrams to be calculated. An extended bibliography on antenna arrays is appended.

Introduction

THROUGHOUT the development of radio communication the engineer has aspired to a directive system whereby radiation might be projected from one point to another with a maximum of efficiency and a minimum of interference with adjacent stations. Also, he has aimed at similar directivity at the receiver to improve the signal-to-noise ratio and otherwise discriminate against un-

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desirable signals. It was recognized at a very early date that directive radio based on wave interference was feasible provided sufficiently short waves could be utilized, and as a result many interesting suggestions to this end were made. However, as is well known, the early development of the radio spectrum proceeded in the direction of long waves rather than short waves, thereby deferring many of the applications of these suggestions.

The principle of wave interference on which most short-wave systems of directive radio are based has probably been known for several centuries. However, the first thorough treatment of this subject was by Sir Thomas Young,[1] who, together with Fresnel, securely established the wave theory of light in the early part of the last century. Even Hooke and Huygens, who had offered the wave theory over a century earlier, failed to recognize the full significance of interference.

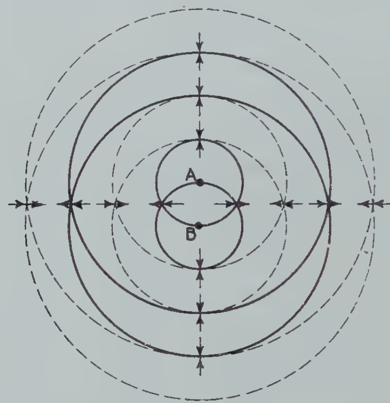
When Hertz started his celebrated experiments to verify Maxwell's theory he was, of course, in full knowledge of these phenomena and their explanation, and invoked their use in proving the existence of electric waves. It is interesting that in some of his experiments he made use of parabolic mirrors for both transmitting and receiving, having directional characteristics very similar to those sometimes used in present day radio practice. It is also of interest that he found that parallel wires stretched over a frame were quite as effective as a reflector as a continuous sheet of metal of similar dimensions, provided the wires were kept parallel to the lines of electric force of the arriving wave. He apparently did not investigate the effect of varying the spacing nor the length of the parallel wires, nor did his subsequent experiments otherwise tend toward the present day antenna array technique.

This paper treats in an elementary way certain aspects of the antenna array problem, principally as regards the manner in which calculated directivity is affected by the number and spacing of the individual antennas which go to make up the array. The theory is applicable only to those forms of directive antennas which may be resolved into a series of individual sources. It does not apply to the so-called wave antenna. However, principles are included which have for some time been in general use in combining two or more such antennas.

Extensive study has been given to directive antenna systems for use in transoceanic radiotelephony. Papers dealing with this general

[1] *Phil. Trans. of Royal Soc.*, **92**, 12, 1802.

subject have appeared from time to time.[2] Further work is in progress. Papers by E. J. Sterba and also by E. E. Bruce and H. T. Friis of the Bell Telephone Laboratories are in preparation which will include certain calculated data similar to those contained in the present paper, and also experimental results obtained from tests on actual antennas of various sizes and proportions.



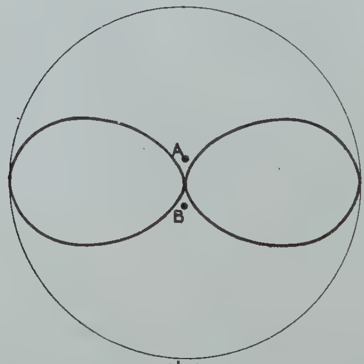


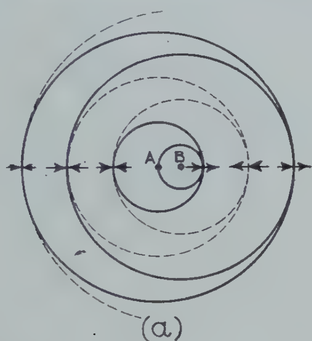
Fig. 1—Interference pattern. Two equiphased sources spaced one-half wavelength.

In the early part of the following discussion each antenna is considered as a spherical source of waves which radiates equal power in all directions. Furthermore, it assumes that the current in each individual source, in a given array, is the same and is not materially

[2] R. M. Foster, "Directive diagrams of antenna arrays," *Bell Sys. Tech. Jour.*, 292, May, 1926. Austin Bailey, S. W. Dean, and W. T. Wintringham, "Receiving system for long wave transatlantic radiotelephony, Proc. I. R. E., **16**, 1694; December, 1928. J. C. Schelleng, "Some problems in short wave radiotelephone transmission," Proc. I. R. E., **18**, 913; June, 1930.

affected in either magnitude or phase by its proximity to other sources. The fair approximation to which these calculated results are realized in practice bespeaks the justification of these assumptions.

The various steps by which present day directional radio has been developed are extremely interesting, but they are so involved in the development of radio itself that their enumeration is considered outside the scope of this paper. However, bibliographies are cited below covering some of their important phases.



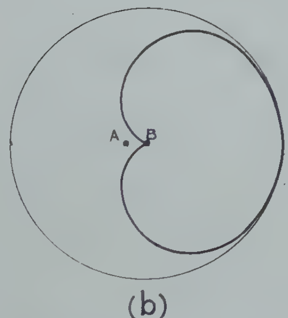


Fig. 2—Interference pattern. Two sources separated in space by one-fourth wavelength and in time by one-fourth period.

Elementary Principles

The interference patterns resulting from a number of individual sources of waves, such as antennas, are dependent on both their spacial arrangement and the magnitudes and relative phases of their forces. This makes possible an almost unlimited number of combinations of which only a portion have thus far found use in communication. This paper will restrict itself mainly to some cases which

are already finding general application. As a suitable introduction to this subject, a very simple case of wave interference is discussed in the following paragraph.

Figs. 1a and 2a depict in a rough way the interference resulting from two independent sources of spherical waves of the same amplitude. In the first case they are spaced 1/2 wavelength but are assumed to be oscillating in phase. In the second case the two sources are separated in space by 1/4 wavelength and in phase by 1/4 period. Crests and troughs are represented respectively by solid and dotted lines. At points where either two crests or two troughs arrive simultaneously the resultant wave is greatly enhanced, whereas at certain other points crests and troughs arrive together, thereby neutralizing each other's effects. At certain intermediate points these interfering effects are only partially complete. Accompanying each figure is a directive diagram (1b and 2b), plotted in polar coordinates, which shows the effectiveness of the wave in each direction. The circle drawn outside each diagram indicates the effect if the radiation had proceeded from a single non-directional source similar to each of the above. The ratio between the areas of the circle and the inscribed diagram gives roughly the power improvement of such a device as manifested in the intensity of the radiated wave. A more exact calculation of this improvement requires an integration of the force components over a unit sphere.

Linear Antenna Arrays

Most directive antenna systems now in general use for short waves may be regarded as special applications of the linear array. This type consists of two or more antennas all having currents of equal amplitude, equispaced along the same straight line. The properties of such arrays have been treated very generally by Foster,[2] whose paper included several hundred directive diagrams, taken in a bisecting plane perpendicular to the axis of each antenna of the array, and typical of the results which may be expected from two antennas and from arrays consisting of 16 antennas. A portion of these diagrams have been reproduced in Figs. 3 and 4 below. The same principles are applicable to both transmission and reception.

In Fig. 3 are shown diagrams resulting from two antennas as the separation is increased from 0 to 1 wavelength in steps of 1/8 wavelength and the phase increased from 0 to 1/2 period in steps of 1/8 period. The line or axis of the array is assumed to be horizontal and the specified phase difference is such that the current in the right-

[2] Loc. cit.

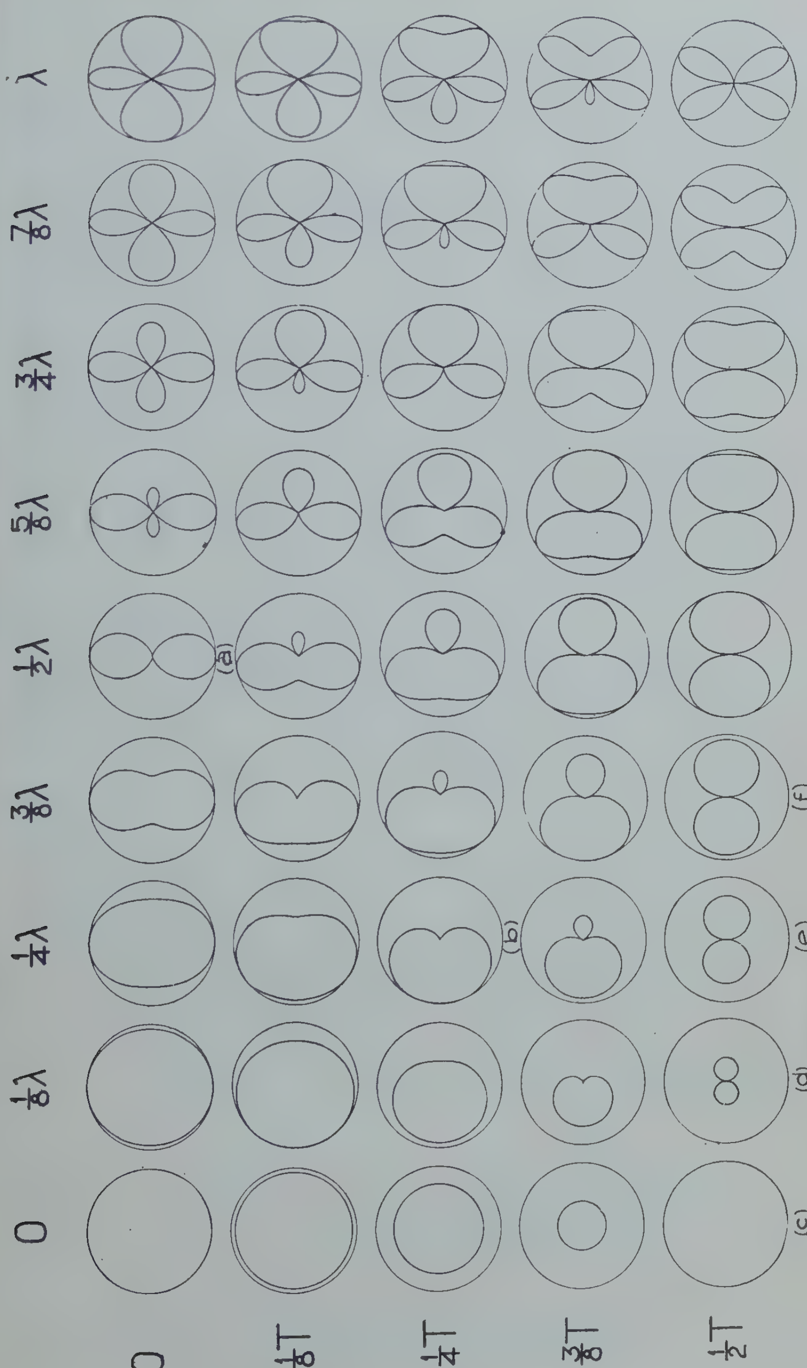


Fig. 3—Directive amplitude diagrams for an array of two antennas. Separation in wavelengths (λ) along the top. Phase difference in periods (T) at the left.

hand antenna is lagging for a transmitting system and leading for a receiver. It will be noted that for phase differences of both 0 and 1/2 T the diagrams are symmetrical about both the horizontal and vertical axes of the figure, whereas for other phases the figures are asymmetrical about the vertical axis except for certain limiting cases. Of these asymmetrical diagrams, that corresponding to phase and spacial separations both of 1/4 (Fig. 3b) is of particular importance and forms the basis of the so-called reflector effect. This particular combination of two sources is referred to later as a unidirectional couplet.[3] In passing it is also of interest to note that the diagram of the coil or frame aerial as generally used is intermediate between Figs. 3c and 3d. Its diagram would not differ essentially from its neighbors, Figs. 3d, 3e, or 3f, except for scale. This scale may conveniently be regarded as a measure of the impedance of the device, or possibly its efficiency, but not necessarily a measure of its usefulness.

Fig. 4 shows similar diagrams resulting from 16 antennas for various phase and space relations. As in Fig. 3, diagrams in the top and bottom rows corresponding respectively to phases of 0 and 1/2 T are symmetrical about both the horizontal and vertical axes. The diagrams in the top row are in general bidirectional, while the bottom row has one bidirectional diagram corresponding to phase and space differences both equal to 1/2. It is of interest that for the most part cases where the phase and space separations are numerically equal correspond to unidirectional diagrams. However, these diagrams are only moderately sharp and thus far such arrays have not been used extensively in practice.

Referring again to the diagrams in the top row corresponding to 16 antennas all driven in phase, we note that directivity becomes progressively sharper as the spacing is increased until in the vicinity of $15/16\,\lambda$ appendages develop which soon surpass in magnitude the desired lobes. This effect is present in the commercial array, and limits, as we shall later see, the gain that may be derived from a given number of elements. The diagrams shown in Fig. 4 for 16 antennas are typical of others where the number of antennas in linear array is fairly large.

The Linear Array and Reflector

One type of array now in commercial use consists of two parallel linear arrays of equiphased elements where the two parallel arrays are spaced 1/4 wavelength and differ in relative phase by 1/4 period. It is

[3] In this, and in other cases in this paper, radiation is referred to as unidirectional when sensibly more power is propagated in one direction than in others.

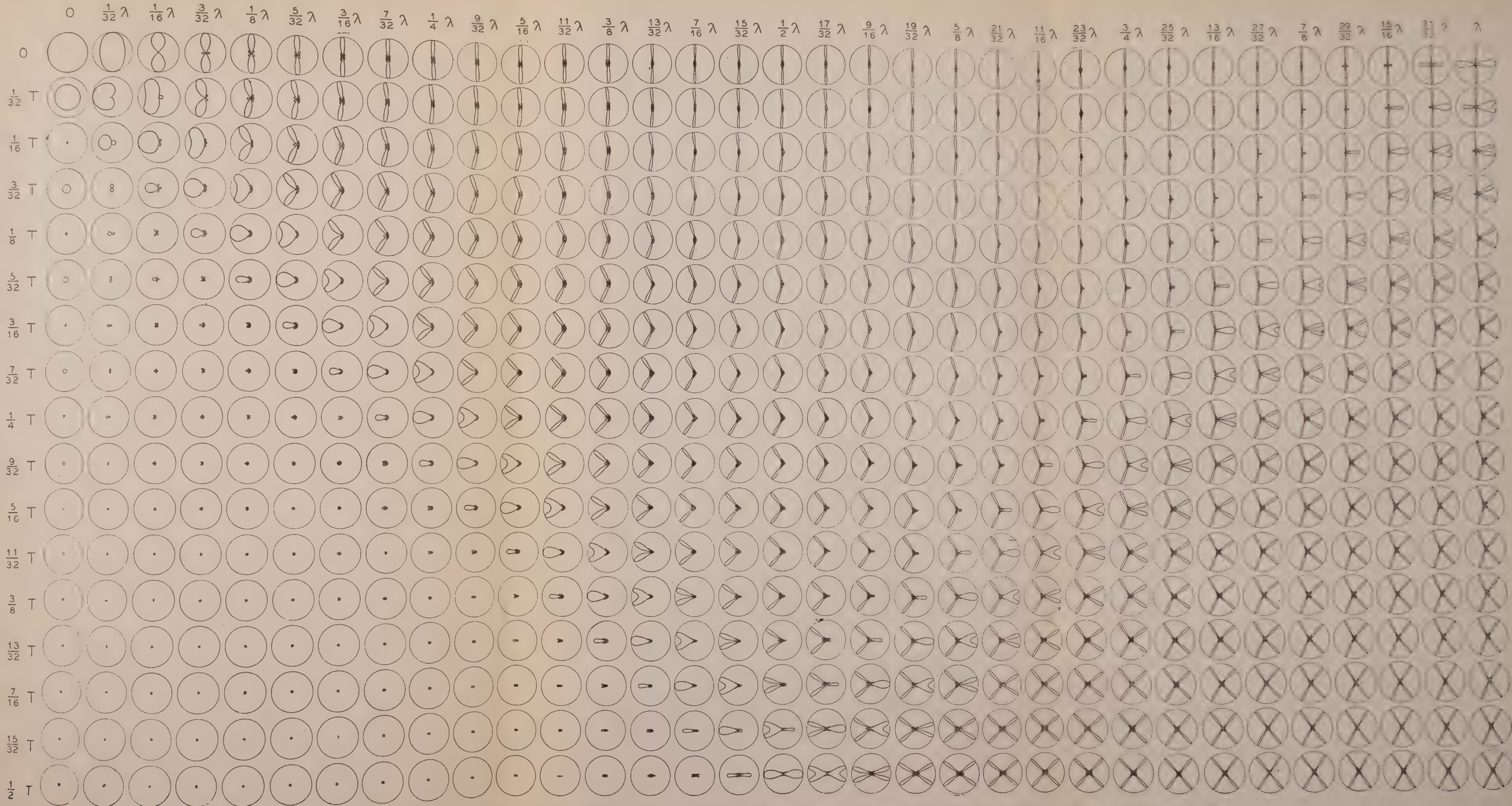


Fig. 4—Directive amplitude diagrams for an array of sixteen antennas. Separation in wave lengths (λ) along the top. Phase difference in periods (T) at the left.

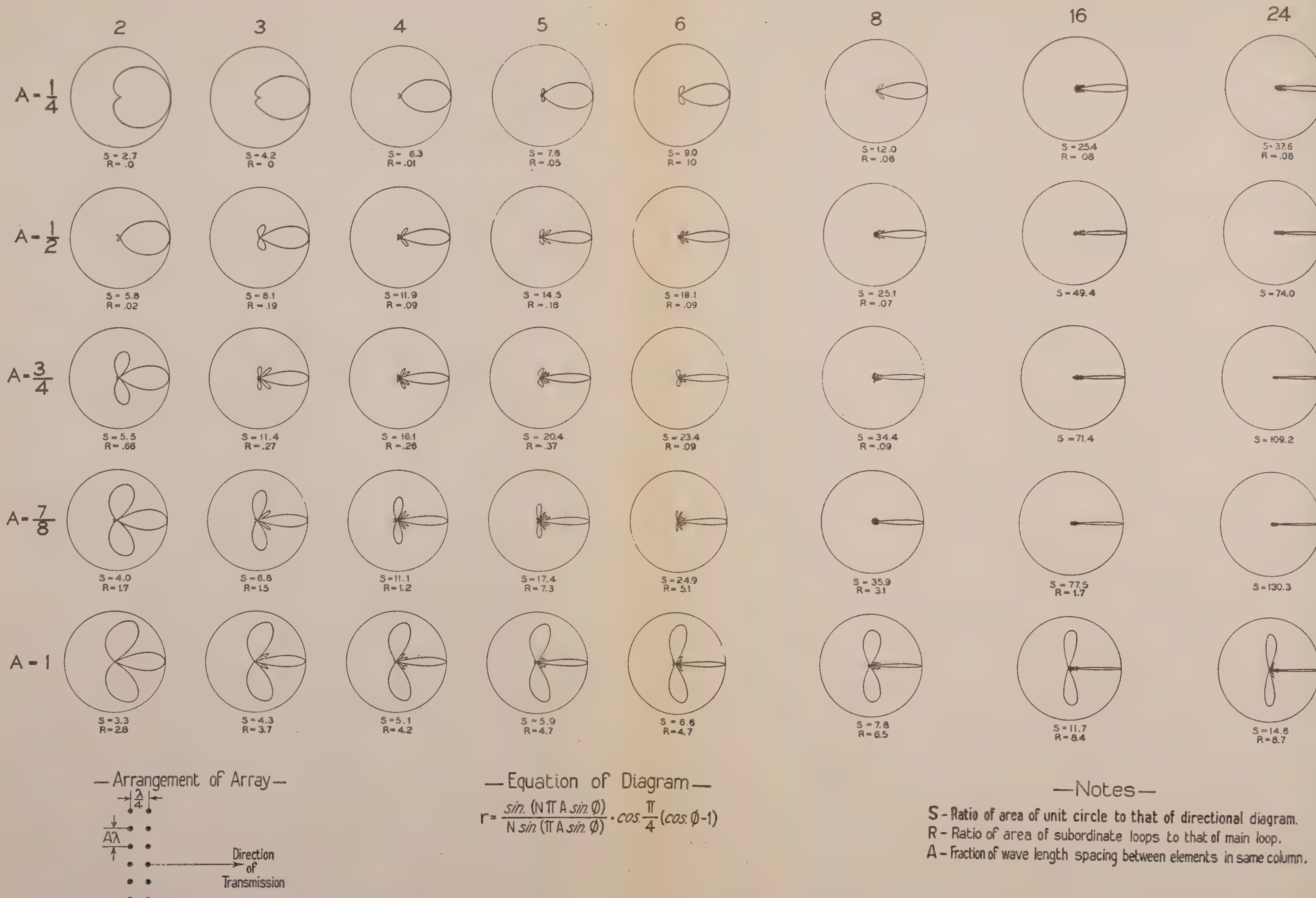


Fig. 5—Horizontal plane diagrams—number of couplets versus separation in wavelengths.

convenient to regard such a device either as two independent linear arrays, each having a directional characteristic as shown in the top row of Fig. 4, or as an array of couplets, each couplet of which has by itself a heart-shaped characteristic. Both antennas of the couplet may be independently driven at their prescribed phase separation of 1/4 period, or one may derive its power from that radiated by the other, in which case the proper phase relation is automatically approximated[4] and the same practical result is obtained. In the latter case one is frequently known as the driven antenna and the other the reflector. This viewpoint is perhaps only a convenience and may not be altogether correct. An array of the above type transmits and receives best in a direction at right angles to its principal dimension. This type is, therefore, frequently known as a broadside array.

Directive Diagrams from Arrays and Reflectors

In Fig. 5 is plotted a series of diagrams in a bisecting plane normal to the axis of each antenna of the array for different broadside arrangements such as are used commercially. They are systematically arranged horizontally in the order of the number of couplets in the array, and vertically with the increased spacing between adjacent couplets.

Several different forms of such directive diagrams are possible, which may be plotted in either polar or rectangular coordinates. In one form all diagrams are roughly of constant area and relative gains from various antenna systems are expressed in terms of the principal radius vector. In the second form the length of the principal radius vector remains constant and the relative gain is roughly inversely proportional to the area of the diagram. The second of these forms has been adopted in this paper largely because of the relative simplicity of the equation of the diagram and the facility with which properties of antennas may be determined.

In the lower left-hand corner of Fig. 5 will be found a plan showing the arrangement of the elements relative to the important direction of transmission. At its right is the general equation of these diagrams. This formula is also given as equation (14) of the appendix where the analytical theory of arrays is developed. Below each diagram is the ratio of the area of the circumscribed unit circle to the area of the horizontal diagram. Here also will be found the ratio of the area of the

[4] The problem of the reflecting antenna has been considered by Wilmotte and McPetrie, *Jour. I. E. E.*, **66**, 949; Englund and Crawford, Proc. I. R. E., **17**, 1277; August, 1928; and Palmer and Honeyball, *Jour. I. E. E.*, **67**, 1045. Their conclusions indicate that the optimum separation between a single antenna and its reflector to give maximum forward radiation is roughly $\lambda/3$. However, it appears that when several antennas and reflectors are involved a separation more nearly $\lambda/4$ is optimum.

subordinate loops to the area of the main loop. The total area may be measured approximately with a planimeter or calculated more accurately by equation (32) in the mathematical appendix. In making up Fig. 5 each diagram was accurately plotted on standard polar coördinate paper from perhaps a hundred calculated points. This was then reduced photographically and the several diagrams were assembled.[5]

Inspection of the diagrams shows that increasing the number of couplets increases in all cases the sharpness of the main loop and hence the gain of the array, However, increasing the separation be-

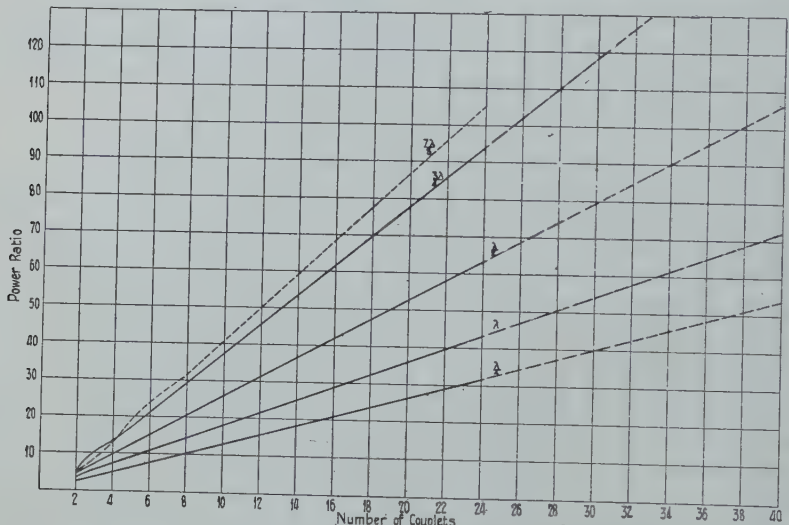


Fig. 6—Antenna arrays. Calculated power ratios vs number of couplets.

tween couplets increases the gain only up to a certain point, after which the formation of parasitic lobes decreases the effectiveness of the array. The trend of these gains may be illustrated more effectively in graphical form.

In Fig. 6 calculated gain ratio is plotted against number of couplets giving one graph for each separation considered. These ratios are not based on the data given in Fig. 5, but were obtained from the integration of the equation of the directional diagram over an arbitrary sphere by use of equation (27) below. It may be noted that for many conditions the difference between these methods of calculating gain is only moderate. These power ratios are for the most part linear,

[5] The diagrams used in this paper were calculated by a group of the Department of Development and Research of the American Telephone and Telegraph Company, under the direction of Miss E. M. Baldwin. Most of the material was checked by Mrs. Isabel Bemis, who assembled it in its present form and prepared the attached bibliography.

indicating that such gains are proportional to the length of the array. This is in keeping with the view that a receiving antenna can intercept wave power more or less in proportion to its dimensions. It is also interesting to note that the slope of the curve of $\lambda/2$ is approximately twice that for $\lambda/4$, so that 16 couplets spaced 1/4 wavelength give approximately the same gain as eight couplets spaced 1/2 wavelength. This again shows that the length of the array is the most important criterion in determining its gain. In Fig. 7 the same data have been plotted in decibels.

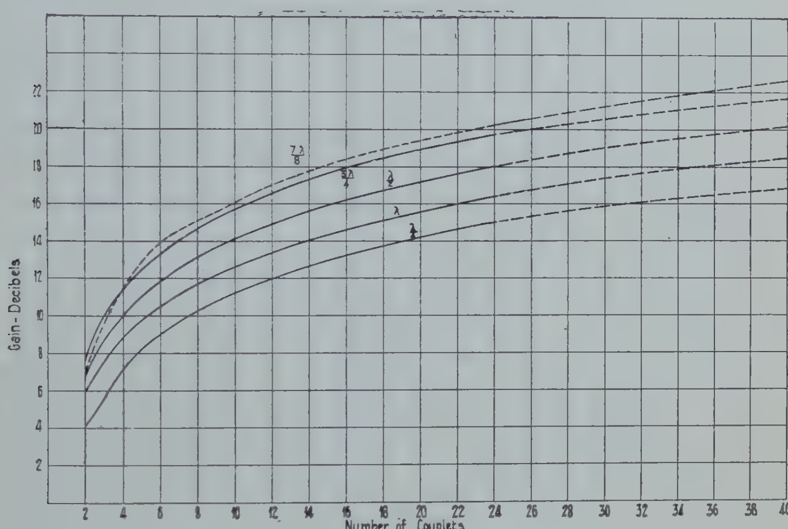


Fig. 7—Antenna arrays. Calculated gains vs number of couplets.

In Fig. 8 gains expressed in decibels are plotted against the separation between elements. This shows more definitely the trend of the antenna gain to a maximum, after which spurious lobes become of importance. Fig. 8 suggests that the spacing, giving optimum gain, would be the desideratum in antenna design. However, this is not necessarily the case, as we shall presently see. It has already been pointed out that the over-all length of array, rather than the spacing or the number of conductors per unit length, constitutes the most important factor in determining the gain. Furthermore, minimum area diagrams are frequently attended by fairly large spurious lobes which are undesirable particularly on receiving antennas. Also the cost of an antenna system of a given height is more or less proportional to its length, and in many cases is not materially affected by the number of conductors present. These considerations, together with the fact that

proper phases may often be most readily accomplished with intervals of either 1/4 wavelength or 1/2 wavelength, have led to a rather general adoption of these closer spacings.

In Fig. 9, approximate gain ratios from arrays of various lengths have been plotted. These are most applicable for separations in the

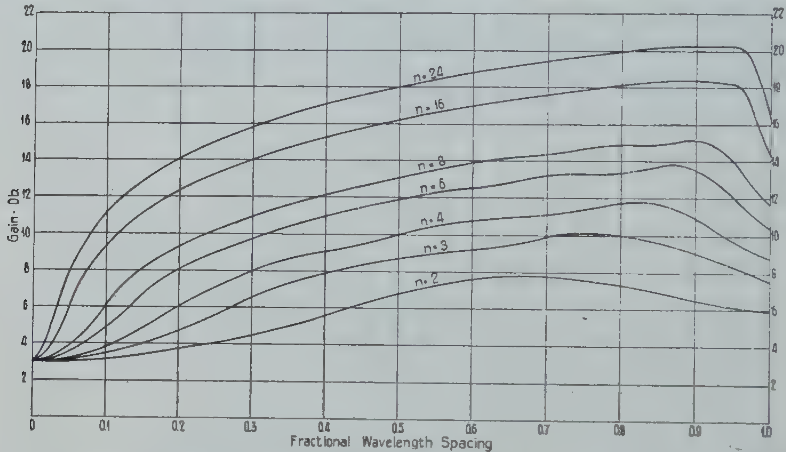


Fig. 8—Antenna arrays. Calculated gains vs lateral spacing between couplets.

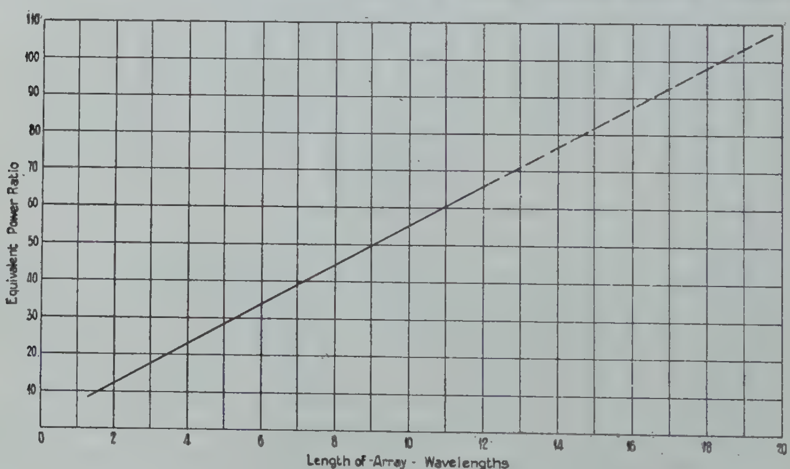


Fig. 9—Approximate gains to be expected from arrays of couplets for spacings of approximately $\lambda/4$ and $\lambda/2$.

vicinity of 1/4 and 1/2 wavelength. Fig. 10 shows the same data plotted in decibels. Within these limits, it appears that the gain ratio may be expressed by the simple formula $G = KL$, where L is the array length in wavelengths and K is approximately 5.6 The result expressed in decibels is $G' = 10 \log_{10}(KL)$.

Measured Antenna Gains

The degree to which the gains calculated above are approximated in practice is indicated by the data given in the diagrams of Figs. 11 and 12 and in Table I.

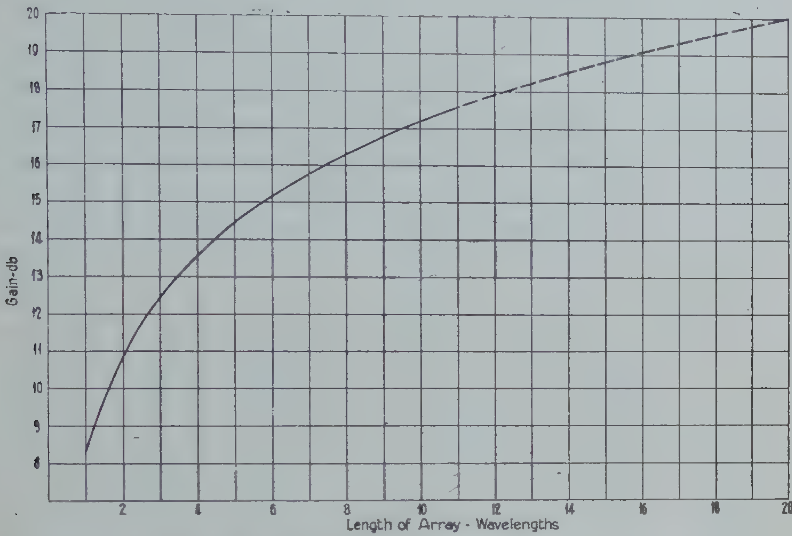


Fig. 10—Approximate gains to be expected from arrays of couplets for spacings of approximately $\lambda/4$ and $\lambda/2$.

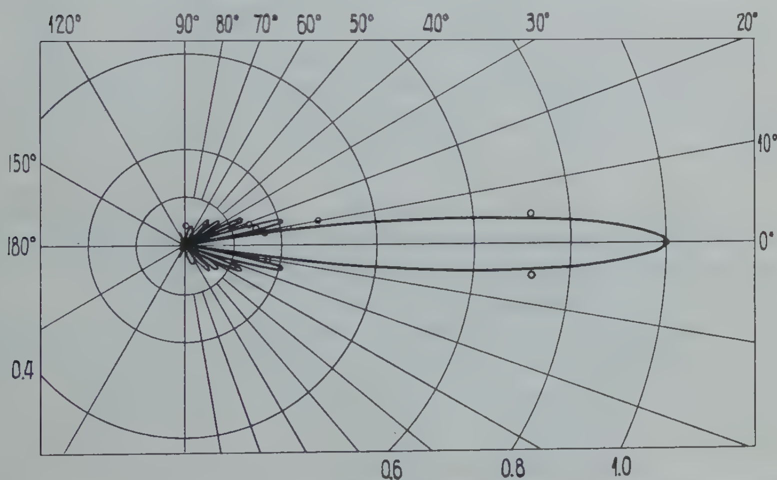


Fig. 11—Calculated directional diagram. Twentyfour couplets spaced one-fourth wavelength. Circles indicate experimental points.

Fig. 11 shows a calculated diagram corresponding to certain receiving arrays used in the transatlantic telephone service between

America and England. Several points are plotted on this diagram which correspond to the relative strengths of signals received at various angles. These points were obtained by observing the relative received signal voltage, measured on a standard field-strength measuring set connected to the array as an electric oscillator of constant amplitude was carried around the array at a distance of perhaps 20 wavelengths. The plotted data correspond to the case where the reflector was "floating." Although this arrangement most nearly corresponds to the conditions assumed in the calculated curve, it is not necessarily the most desirable adjustment to minimize noise arriving from the rear. This diagram corresponds to the antennas designated as 1-A, 2-A, and 3-A in Table I. These antennas consist effectively of 24 vertical couplets spaced horizontally at intervals of 1/4 wavelength.

TABLE I

Array Designation	Nominal Operating Frequency Megacycles	Number Couplets	Spacing	Measured Gain Over Similar Single Element db	Calculated Gain db	Difference db
1–A	18	24	$\lambda/4$	15.3	15.0	+0.3
2–A	18	24	$\lambda/4$	15.2	15.0	+0.2
3–A	18	24	$\lambda/4$	15.0	15.0	0.0
1–B	12	24	$\lambda/4$	15.6	15.0	+0.6
2–B	12	24	$\lambda/4$	14.5	15.0	−0.5
3–B	15	24	$\lambda/4$	13.6	15.0	−1.4
4–B	15	24	$\lambda/4$	16.6	15.0	+1.6
2–C	10	24	$\lambda/4$	16.3	15.0	+1.3
3–C	10	24	$\lambda/4$	15.5	15.0	+0.5
1–C	9	18	$\lambda/4$	13.6	13.8	−0.2
D*	14	9	$\lambda/2$	13.0	13.7	−0.7

* This antenna actually consisted of two arrays of four couplets each spaced laterally by one wavelength. The resultant diagram of such an array is for all practical purposes the same as that produced by a continuous array of nine couplets.

In this table are given further data on the strength of signals received on arrays, as compared with those received simultaneously on a single element of similar structure and height above earth. The different antennas represented involve varying conditions of wavelength, height above earth, adjacent terrain, and types of support. These details are not believed to be of sufficient importance for discussion here. Two different array lengths are represented. The relative gains were substantially the same when observed on a local source of waves and when the signal came from a distant station. The last array represented in Table I was one used for transmitting. To effect the test, equal power was transmitted alternately from the array and from a single element while comparative measurements of electric field strength were made at a distance of approximately 3500 miles.

The datum given is the mean of perhaps 100 observations extending over a total of eight hours on three different days. Two errors are involved in the data of Table I. One is due to the doubtful magnitude of a correction necessary to account for the various heights at which the arrays were located above the earth and the second is the error of measurement of gain as compared with the reference antenna. These errors are approximately equal and together amount to ± 1 db.

In order to test further the agreement between measured gains and those calculated from the simple assumptions above, a receiving array was assembled step by step and corresponding measurements made.

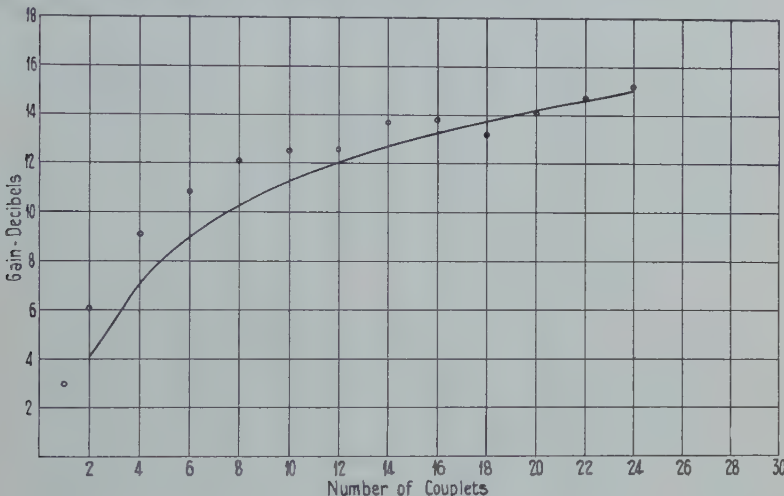


Fig. 12—Relation of measured to calculated gain of receiving antenna array at 14,350 kc.

Certain precautions, such as to maintain impedance matches at points of coupling, were observed. The resulting data were plotted as points in Fig. 12. A smooth curve represents the corresponding calculated data. It will be observed that the measured values are consistently higher than those calculated at the lower end of the curve, and in this region the agreement can hardly be regarded as satisfactory. However, limited time prevented a thorough study of the errors of measurement. Consequently these limited data may not be regarded as any adequate test of the theory.

Combinations of Arrays

It may be shown that two or more similar directive systems may be combined to give a total directive effect, represented by the product of the individual effect, multiplied by the group effect. This principle

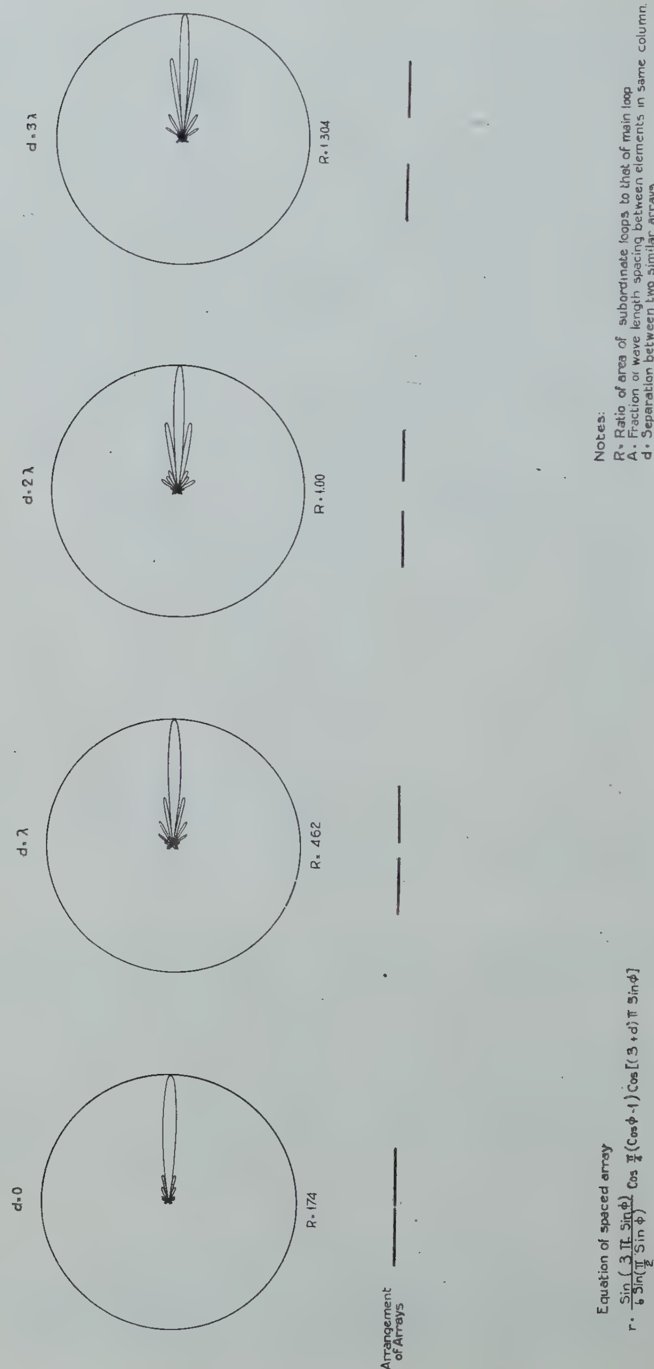


Fig. 13—Directional diagrams in horizontal plane of two arrays spaced laterally as noted. Case N=6 and A=½.

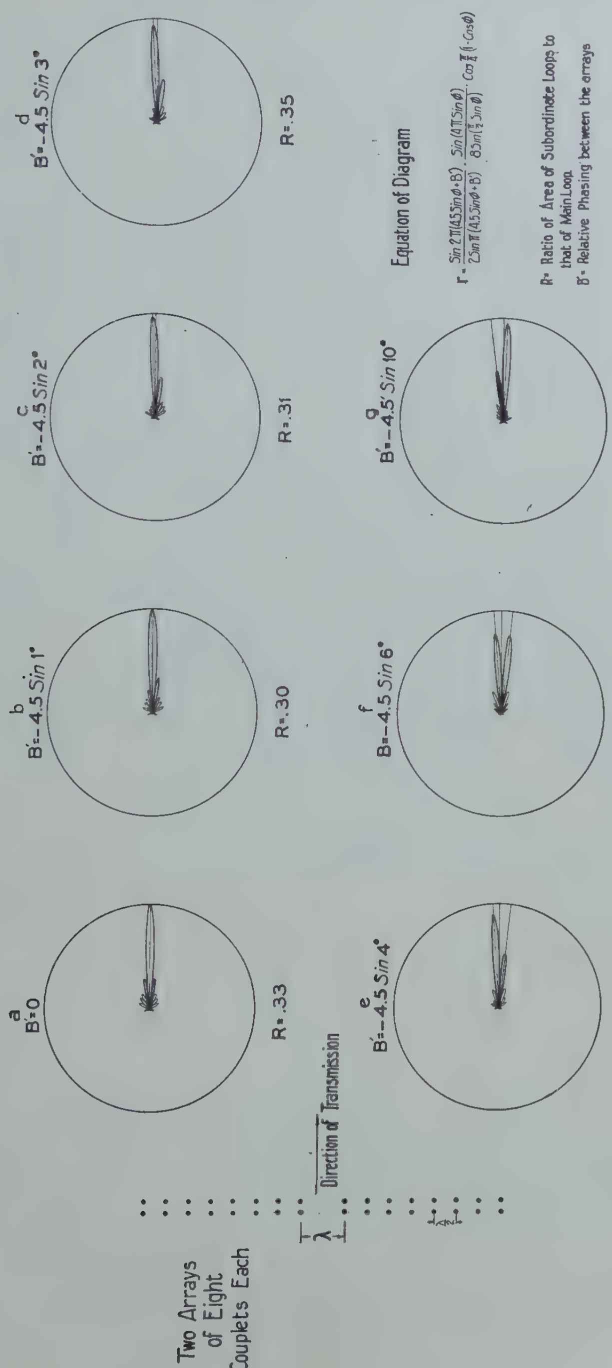


Fig. 14—Effect of phasing between two arrays.

is partially covered by equation (35) of the mathematical appendix. Two cases are of special interest. First, it is sometimes desirable to divide an array into two or more bays, in order to make room for a supporting structure. This, of course, gives rise to a definite discontinuity in the over-all array.

Fig. 13 shows a series of diagrams resulting from a typical case of two such arrays, each having a length of 2-1/2 wavelengths but separated variously from 0 to 2 wavelengths in steps as noted. These diagrams, of course, do not take into consideration the reaction resulting from proximity to an antenna mast, located in such an opening. The most important result is to emphasize the spurious lobes, as the spacing between arrays is increased.

A second effect of grouping which is of considerable interest is that of varying the direction of transmission by altering the respective phases between two or more arrays or between sections of the same array. In Fig. 14 a series of diagrams is shown for a typical case of two 3-1/2 wavelength arrays, spaced one wavelength. All elements in the same array are driven in phase, but the two arrays differ in phase by various amounts, as noted. It will be observed that the possible rotational effect is very limited. The general equation for this diagram is given by formula (36) of the mathematical appendix.

This effect was investigated further by assuming a continuous array 7-1/2 wavelengths long, made up of 16 couplets spaced at intervals of 1/2 wavelength. The results are depicted in Fig. 15. The top row assumes that the array is divided into two sections of eight couplets each. This gives similar but not exactly the same results as those of Fig. 14. The array, however, might have been divided into other sections for purposes of phasing. The various possible combinations are tabulated below:

Number of Sections	Number of Couplets per Section
2	8
4	4
8	2
16	1

Diagrams in rows two, three, and four show that, as the array continues to be divided into smaller sections, the direction of transmission is capable of greater variation without sensible loss of sharpness. If the array be divided into two sections this range is limited to perhaps 3 deg. as in the case depicted in Fig. 14. Although this i very moderate, it is extremely useful in correcting for any errors in the orientation of the supporting structure or possibly correcting for deviation of the projected radiation caused by peculiarities of the adjacent terrain.

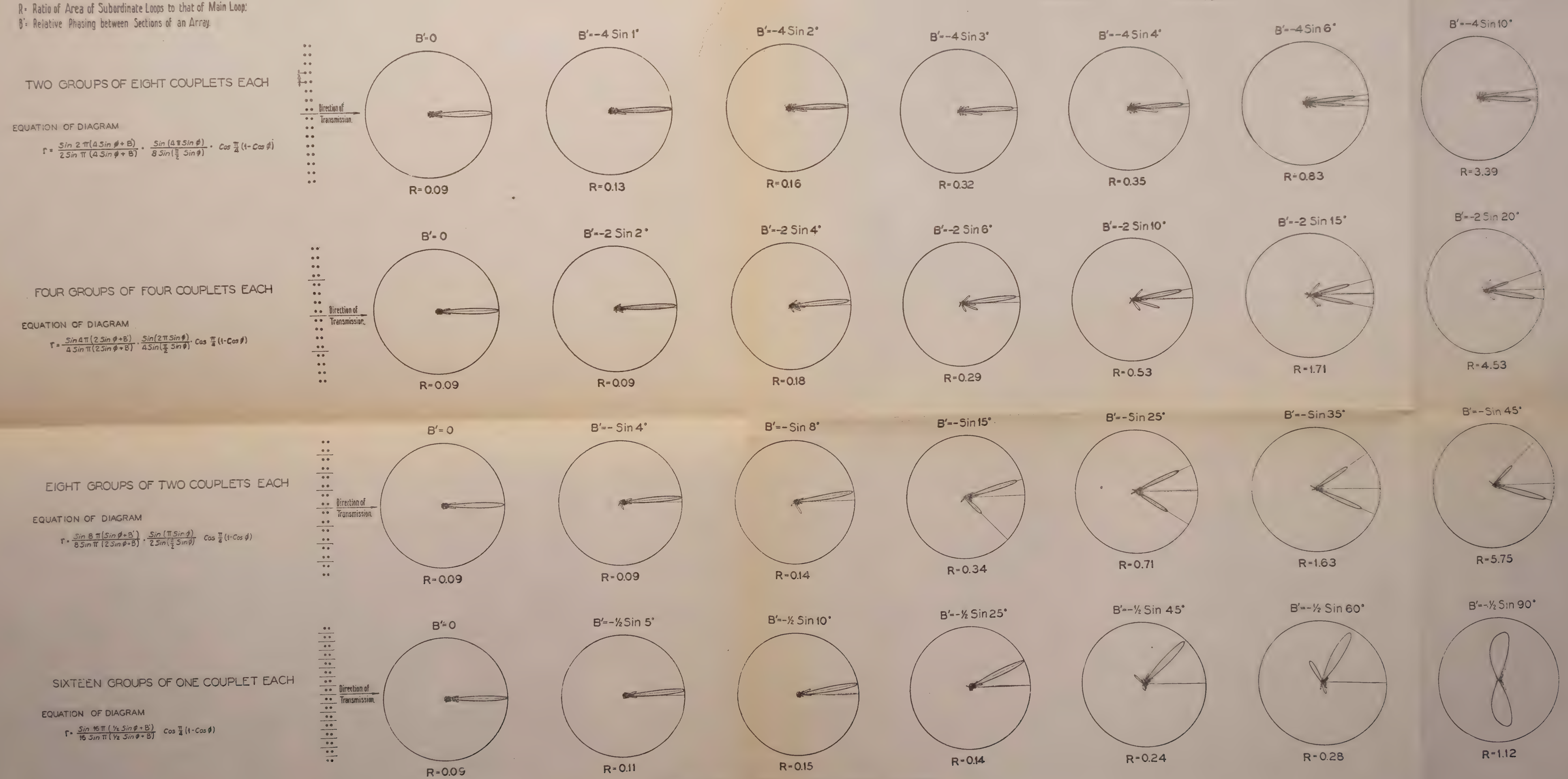


Fig. 15—Effect of phasing between sections of an array.

If the array is divided into four sections the rotation may extend over a range of perhaps 9 deg., while for eight sections it may be 15 deg. The final case of 16 sections of one couplet each permits of considerable flexibility such as would be useful in operating with several distant stations in the same general direction. It should be pointed out, however, that the problem of making 16 phase adjustments each time a station wishes to change its direction of transmission is of considerable magnitude. For the particular case illustrated above it appears that the maximum rotation of the projected radiation is more or less proportional to the number of sections into which the array is divided. It may readily be seen from the two top rows of diagrams in Fig. 15 that continued addition of phasing amounts effectively to negative rotation. This may also be seen from an analysis of the equation of the diagram.

Fields of Linear Arrays

The successful use of an array of couplets to give unidirectivity suggests that the use of more than two parallel linear arrays might further be employed to advantage.[6] Obviously many such combinations are possible, but one of some interest has been investigated below. As a concrete example of this variation of gain with arrangement of arrays, a series of diagrams for 36 elements has been plotted in Fig. 16. The condition of spacing and phase intervals between columns of each of $1/4\lambda$ has been chosen. The horizontal characteristic is given for separations between rows of both 1/2 and 1/4 wavelength. The vertical characteristic common to these two separations is also shown. The equation of the diagram is given in formula (17) of the mathematical appendix below.

It will be observed from Fig. 16 that the horizontal directivity is for the most part only moderate, but approaches a maximum for the condition where a long broadside array prevails, whereas the vertical directivity is increased by increasing the number of columns in the field. A substantial loop will be found near the rear of diagrams corresponding to an odd number of columns. It is of further interest that, as far as horizontal directivity alone is concerned, the optimum may be derived either from a single array of 36 elements or from 18 couplets. Considerations of both minimum interference and total gain, however, make the latter preferable. These conclusions may also be reached by more direct analysis.[7]

[6] U. S. Patent 1,643,323, John Stone Stone, September 27, 1927.

[7] Wilmotte, "General considerations of the directivity of beam systems *Jour. I. E. E.*, **66**, 955.

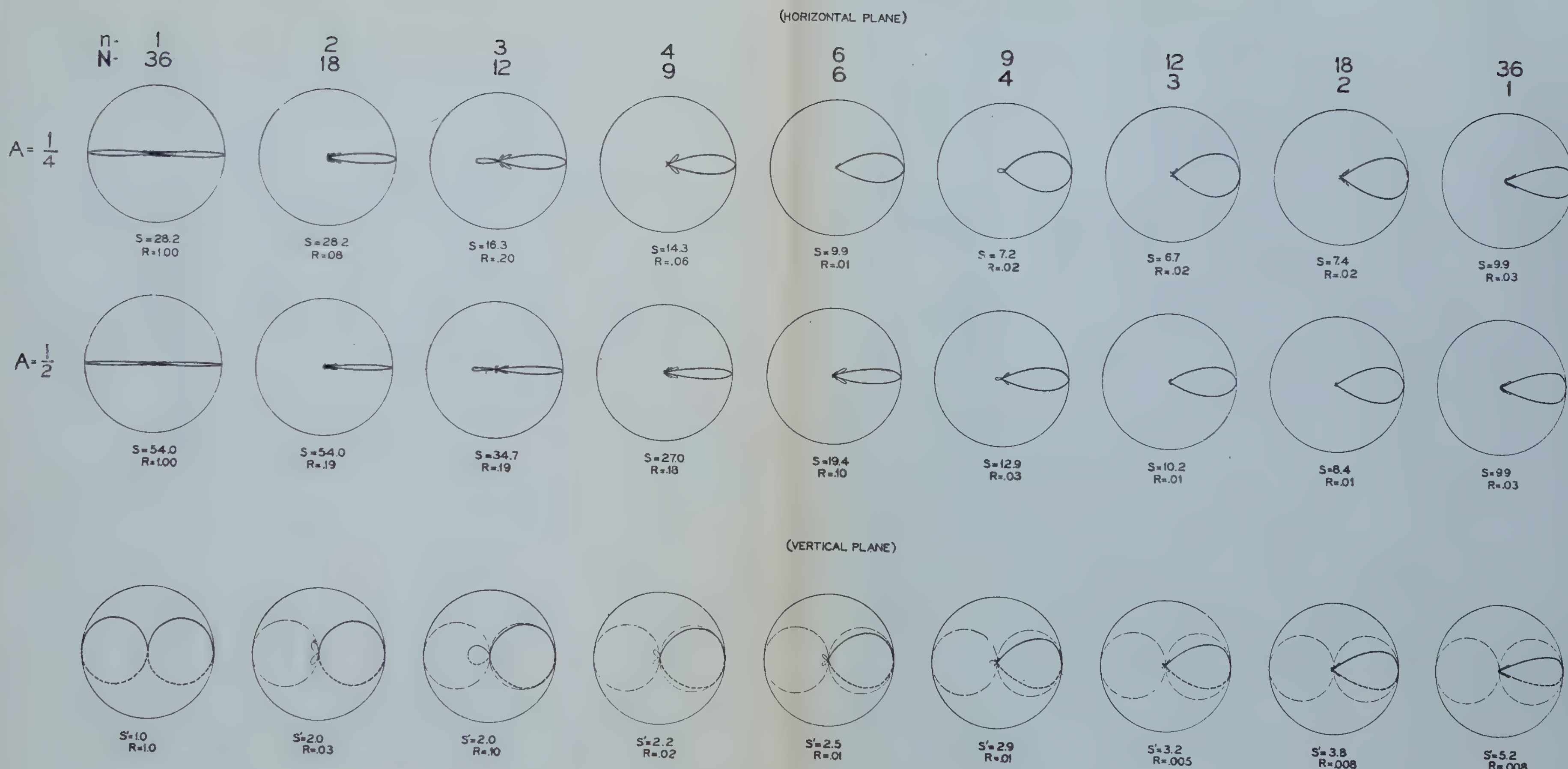
Stacked Antennas

Thus far the discussion has centered mainly around directivity produced by placing vertical antennas in horizontal array. Added gain may be had also by incorporating directivity in a vertical plane.[8] This is frequently accomplished by arranging individual antennas one above another with their axes collinear, and is sometimes known as stacking. The fundamental principles of analysis are the same as those already utilized. However, an approximate correction must be allowed to account for the fact that the radiation from a linear oscillator increases from zero along the axis to a maximum in a plane perpendicular to the axis. The directional characteristic in planes passed through and parallel to such a radiator is approximated by two tangent circles.

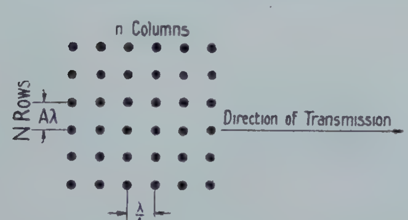
Fig. 17 shows a series of directional diagrams indicating the results of stacking unidirectional couplets. The diagrams shown refer to the plane passed through the axes of the two linear oscillators comprising the couplet. On each diagram is a unit circle corresponding to a single point source. Inscribed are the two tangent circles, representing the vertical directional characteristic of a single linear source. Inside one of the tangent circles is the final directional diagram of the stacked array. The ratio of the area of the tangent circles to that of the characteristic diagram is given under each figure. This may be regarded as a rough measure of the relative gain. These diagrams are arranged horizontally in order of increasing number of couplets and vertically in order of separation. It frequently happens in practice that each radiator is approximately 1/2 wavelength long so it is convenient to utilize a vertical spacing interval also of 1/2 wavelength. Consequently the second row of diagrams is probably of greatest practical interest. In calculating these diagrams earth effects have been ignored.

In Figs. 18 and 19, the gain in decibels to be expected from stacking couplets has been plotted against number of couplets and fractional wavelength spacing. These values, like those for Figs. 7 and 8 above, were calculated by integrating the equation of diagram over a sphere of arbitrary radius. This was accomplished by use of equation (30) below. On account of the limited data at hand, Figs. 18 and 19 should be regarded only as a convenient method of illustrating the trend of the variables. These indicate that somewhat lower corresponding improvements result from stacking than from increasing the length of an array.

[8] U. S. Patent 1,683,739, John Stone Stone, September 11, 1928.



ARRANGEMENT OF ARRAY



EQUATIONS OF DIAGRAMS

$$r_{\theta=0} = \frac{\sin(N\pi A \sin\phi)}{N\sin(\pi A \sin\phi)} \cdot \frac{\sin\left(n\frac{\pi}{4}[\cos\phi - 1]\right)}{n\sin\left(\frac{\pi}{4}[\cos\phi - 1]\right)}$$

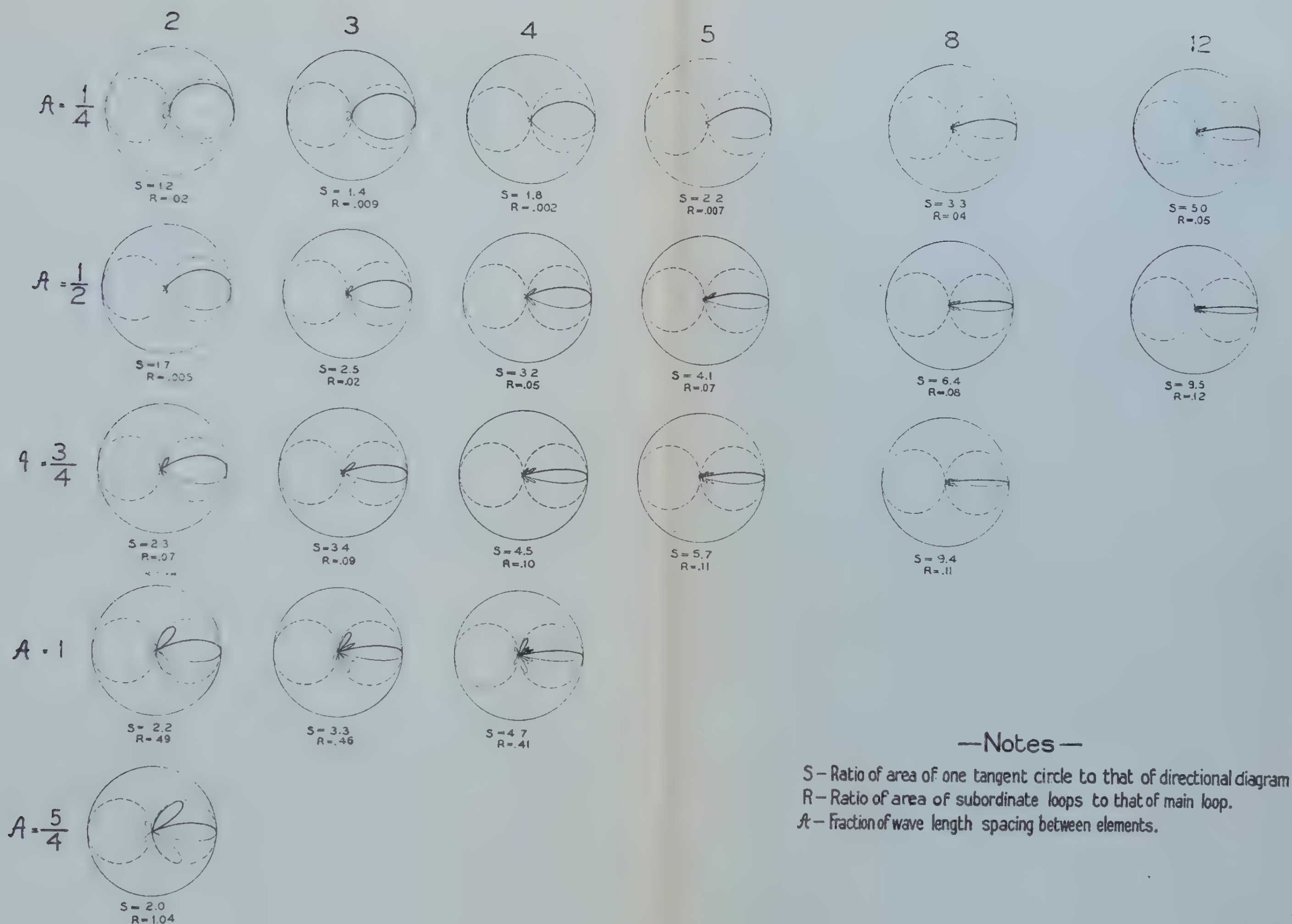
$$r_{\phi=0} = \frac{\sin(N\pi A \sin\theta)}{N\sin(\pi A \sin\theta)} \cdot \frac{\sin\left(n\frac{\pi}{4}[\sin\theta - 1]\right)}{n\sin\left(\frac{\pi}{4}[\sin\theta - 1]\right)} \cdot \sin\theta$$

$$r_{\phi=180^\circ} = \frac{\sin(N\pi A \sin\theta)}{N\sin(\pi A \sin\theta)} \cdot \frac{\sin\left(n\frac{\pi}{4}[\sin\theta + 1]\right)}{n\sin\left(\frac{\pi}{4}[\sin\theta + 1]\right)} \sin\theta$$

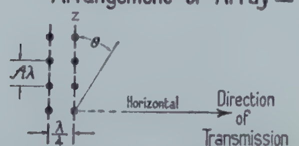
NOTES

S. Ratio of area of unit circle to that of directional diagram.
S'. Ratio of area of tangent circles to that of directional diagram
R. Ratio of area of subordinate loops to that of main loop.
A. Fraction of wave length spacing between elements in same column.

Fig. 16—Directional diagrams due to a field of thirty-six antennas.



—Arrangement of Array—



—Equations of Diagram—

$$\Gamma_{\phi=0°} = \frac{\sin(n\pi A\cos\theta)}{n\sin(\pi A\cos\theta)} \cdot \cos\frac{\pi}{4}(\sin\theta-1)\sin\theta$$

$$\Gamma_{\phi=180°} = \frac{\sin(n\pi A\cos\theta)}{n\sin(\pi A\cos\theta)} \cdot \cos\frac{\pi}{4}(\sin\theta+1)\sin\theta$$

Fig. 17—Vertical plane diagrams due to couplets of coaxial antennas—number of couplets versus separation in wavelengths.

Arrays Incorporating Both Horizontal and Vertical Directivity

The gains of arrays combining both horizontal and vertical directivity may not be simply calculated by adding the gains (expressed in decibels) corresponding to elements arranged respectively along the

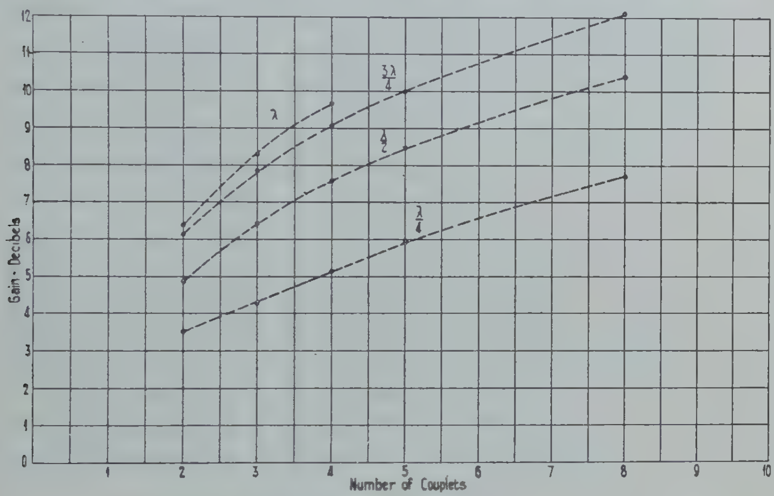


Fig. 18—Calculated gains from stacked antennas.

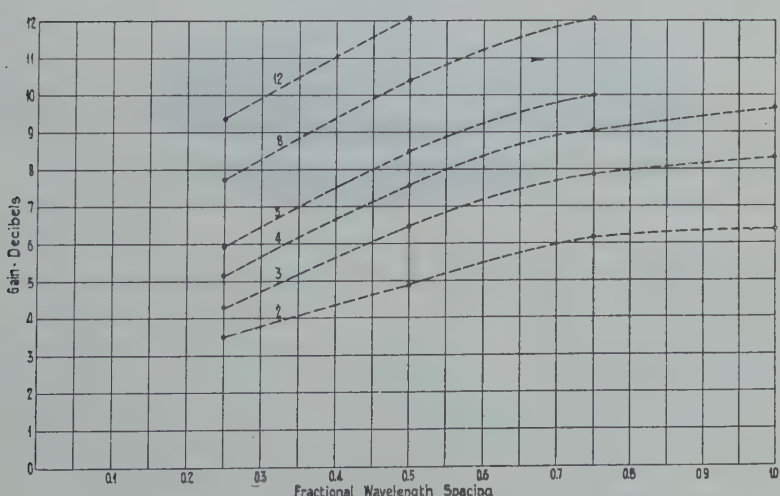


Fig. 19—Calculated gains from stacked antennas.

two principal coördinate axes. However, they may be calculated except for earth effects by means of equation (26) below. Some calculations of this kind have been made and the data are tabulated below.

They assume a total of 36 couplets which are arranged variously as noted. In the first case all 36 couplets are arranged as a simple horizontal array. The second case assumes that they are arranged in a broadside rectangle two elements high and 18 elements wide. This combination may be regarded as two arrays of 18 couplets arranged one

TABLE II

Number of Couplets Along Horizontal Axis	Number of Couplets Along Vertical Axis	Gain over Single Half-Wave Element Decibels
N	η	G
36	1	19.7
18	2	19.0
12	3	18.9
9	4	18.8
6	6	18.7
4	9	18.6
1	36	17.5

above the other. The third case similarly assumes three arrays of 12 couplets each. A separation between couplets of 1/2 wavelength has been assumed throughout. The most economical arrangement of such an array depends not only on the relative costs of real estate and towers, but also on feed-line losses and effects due to the proximity of the earth. The latter have specifically been omitted in this discussion.

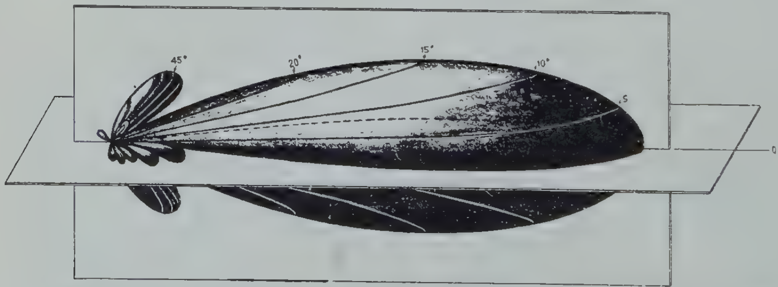


Fig. 20—Approximate three-dimensional diagram. Linear antenna array with reflector. Aperture two wavelengths by eight wavelengths.

Fig. 20 shows roughly the calculated directional characteristics of a typical stacked array incorporating both horizontal and vertical directivity. The planes passed through the diagram serve only as convenient references to assist in visualizing the horizontal and vertical diagrams. Earth effects, of course, have been ignored.

Appendix

A general case of linear arrays, which includes those used extensively in short-wave radio work, consists of a number of sources equispaced and equiphased along each of the three principal coordinate axes such that the space between sources is made up of rectangular parallelopipeds with the individual sources located at each corner.

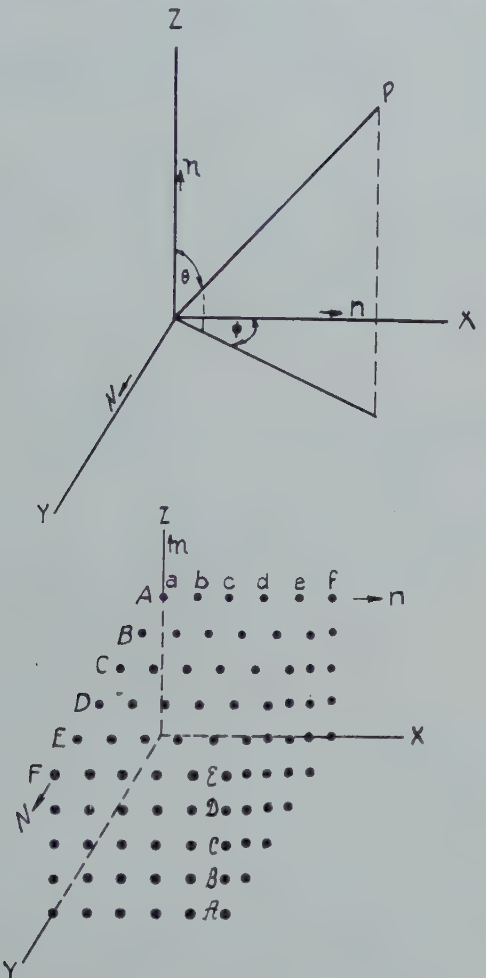


Fig. 21—General case of linear antenna arrays.

This may be regarded as $\mathcal{N}$ parallel planes each made up of N paralell columns where each column is made up of n individual radiating elements. The arrangement is made more evident by Fig. 21. The usual conventions for representing three-dimensional space have been

adopted. We may designate the spacing between elements along the x, y, and z axes, respectively, by $a\lambda$, $A\lambda$, and $\mathcal{A}\lambda$ and their corresponding phase displacements between adjacent elements along the three principal axes by bT, BT and $\mathcal{B}T$.

The distance from any point in space to a particular radiator is

$$R_{nN\eta} \doteq R - (\mathcal{N} - 1)\mathcal{A}\lambda \cos\theta - (N - 1)A\lambda \cos\phi \sin\theta \\ - (n - 1)a\lambda \sin\theta \sin\phi. \tag{1}$$

Similarly the time phase of any particular element relative to the origin is

$$\delta_{nN\eta} = [(n - 1)b + (N - 1)B + (\mathcal{N}-1)\mathcal{B}]T. \tag{2}$$

The instantaneous value of the electric field at any remote point P due to one of these sources is given by

$$E_{n'} = A \cos\frac{2\pi}{\lambda}(ct - R_{n'}) + \delta_{n'} = A \cos\psi_{n'} \tag{3}$$

where $n' = nN$

The resultant interfering effect at a point P due to n' such sources all of equal amplitude is given by

$$E^2 = n'E_0^2 + 2E_0^2[\cos(\psi_1 - \psi_2) + \cos(\psi_1 - \psi_3) + \cos(\psi_1 - \psi_4) + \ldots \text{etc.} \\ + \cos(\psi_2 - \psi_3) + \cos(\psi_2 - \psi_4) + \cos(\psi_2 - \psi_5) + \ldots \text{etc} \\ + \cos(\psi_3 - \psi_4) + \cos(\psi_3 - \psi_5) + \ldots \ldots \ldots \text{etc.} \\ + \cos(\psi_{n'-1} - \psi_{n'})]. \tag{4}$$

The summation above gives rise to three series as follows:

$$S_x = (n - 1)\cos 2\pi(a \sin\theta \cdot \sin\phi + b) + (n - 2)\cos 2\cdot 2\pi(a \sin\theta \\ \cdot \sin\phi + b) + (n - 3)\cos 3\cdot 2\pi(a \sin\theta \cdot \sin\phi + b) + \ldots \\ + \cos(n - 1)\cdot 2\pi(a \sin\theta \cdot \sin\phi + b) \tag{5}$$

$$S_y = (N - 1)\cos 2\pi(A \sin\theta \cdot \cos\phi + B) + (N - 2)\cos 2\cdot 2\pi(A \sin\theta \\ \cdot \cos\phi + B) + (N - 3)\cos 3\cdot 2\pi(A \sin\theta \cdot \cos\phi + B) + \ldots \\ + \cos(N - 1)\cdot 2\pi(A \sin\theta \cdot \cos\phi + B) \tag{6}$$

$$S_z = (\mathcal{N} - 1)\cos 2\pi(\mathcal{A}\cos\theta + \mathcal{B}) + (\mathcal{N} - 2)\cos 2\cdot 2\pi(\mathcal{A}\cos\theta + \mathcal{B}) \\ + (\mathcal{N} - 3)\cos 3\cdot 2\pi(\mathcal{A}\cos\theta + \mathcal{B}) + \ldots + \cos(\mathcal{N} - 1)\cdot 2\pi \\ (\mathcal{A}\cos\theta + \mathcal{B}) \tag{7}$$

such that

$$E^2 = E_0^2(n + 2S_x)(N + 2S_y)(\mathcal{N} + 2S_z) \tag{8}$$

Each series is of the type

$$S = (n-1)\cos x + (n-2)\cos 2x + (n-3)\cos 3x + \ldots + \cos(n-1)x \tag{9}$$

which is readily summed giving

$$n + 2S = \frac{(\cos nx - 1)}{(\cos x - 1)} = \frac{\sin^2 \frac{nx}{2}}{\sin^2 \frac{x}{2}} \tag{10}$$

so

$$E = E_0 \frac{\sin n\pi(a \cos\phi \cdot \sin\theta + b)}{\sin\pi(a\cos\phi\cdot\sin\theta + b)} \cdot \frac{\sin N\pi(A \sin\phi\cdot\sin\theta + B)}{\sin\pi(A\sin\phi\cdot\sin\theta + B)} \cdot \frac{\sin\mathcal{N}\pi(\mathcal{A}\cos\theta + \mathcal{B})}{\sin\pi(\mathcal{A}\cos\theta + \mathcal{B})} \tag{11}$$

Reducing to common voltage level and including a term $\sin\theta$ to cover the case of radiation from linear oscillators we have for the equation of the directional diagram

$$r = \frac{\sin n\pi(a\cos\phi\cdot\sin\theta + b)}{n\sin\pi(a\cos\phi\cdot\sin\theta + b)} \cdot \frac{\sin N\pi(A\sin\phi\cdot\sin\theta + B)}{N\sin\pi(A\sin\phi\cdot\sin\theta + B)} \cdot \frac{\sin\mathcal{N}\pi(\mathcal{A}\cos\theta + \mathcal{B})}{\mathcal{N}\sin\pi(\mathcal{A}\cos\theta + \mathcal{B})}\sin\theta \tag{12}$$

It will be recognized that this equation is made up of four factors. The first three account for the effects of the disposition of elements along the x, y, and z axes, respectively, while the fourth, of course, accounts for the direction of radiation from a linear oscillator. This is an equation giving magnitudes only. In plotting polar diagrams from this equation negative signs have no physical significance, and are plotted in a positive sense.

An examination of this equation shows that there are many possibilities which allow radiation in preferred directions, and at the same time limit it in others. Some of these are discussed below.

Special Cases

If we assume $n=2$, $a=\frac{1}{4}$, $b=-\frac{1}{4}$ and $B=\mathcal{B}=0$

$$r = \frac{\sin\,(N\pi A \sin\phi\cdot\sin\theta)}{N\sin\,(\pi A\sin\phi\cdot\sin\theta)}\cdot\frac{\sin\,(\mathcal{N}\pi\mathcal{A}\cos\theta)}{\mathcal{N}\sin\,(\pi\mathcal{A}\cos\theta)}\cdot\cos\frac{\pi}{4}(\cos\phi\cdot\sin\theta-1)\cdot\sin\theta. \tag{13}$$

This corresponds to the practical case of transmission along the x axis from an antenna curtain and reflector made up of N vertical columns of $\mathcal{N}$ elements each.

The equation for the diagram in the (XY) plane may be had by placing $\theta=\pi/2$ giving

$$r = \frac{\sin\,(N\pi A\sin\phi)}{N\sin\,(\pi A\sin\phi)}\cos\frac{\pi}{4}(\cos\phi-1) \tag{14}$$

which is the equation of the diagrams in Fig. 5 above. The corresponding equation for the principal vertical section may be had by placing $\phi=0$ and $\phi=\pi$ giving

$$\left.\begin{aligned} r &= \frac{\sin\,(\mathcal{N}\pi\mathcal{A}\cos\theta)}{\mathcal{N}\sin\,(\pi\mathcal{A}\cos\theta)}\cos\frac{\pi}{4}(\sin\theta-1)\sin\theta \\ \text{and}\quad r &= \frac{\sin\,(\mathcal{N}\pi\mathcal{A}\cos\theta)}{\mathcal{N}\sin\,(\pi\mathcal{A}\cos\theta)}\cos\frac{\pi}{4}(\sin\theta+1)\sin\theta \end{aligned}\right\} \tag{15}$$

which is the equation for the diagrams of Fig. 17.

The diagram of a single linear array of point sources is specified by the first term of equation (12) where $\theta=\pi/2$ or

$$r = \frac{\sin n\pi(a\cos\phi+b)}{n\sin\pi(a\cos\phi+b)} \tag{16}$$

The diagrams of Figs. 3 and 4 above may be calculated from equation (16) by placing $n=2$ and $n=16$, respectively. This also agrees with Foster's equation (1), page 307.[2]

The diagram of a field of coplanar linear arrays such as depicted in Fig. 16 above follows from equation (12) by placing $\mathcal{N}=1$, $a=\frac{1}{4}$, $b=-\frac{1}{4}$ and $B=0$.

If the diagram is to be restricted to the (XY) plane, $\theta=\pi/2$ and

$$r = \frac{\sin\,(N\pi A\sin\phi)}{N\sin\,(\pi A\sin\phi)}\cdot\frac{\sin\left(n\frac{\pi}{4}(\cos\phi-1)\right)}{n\sin\left(\frac{\pi}{4}(\cos\phi-1)\right)} \tag{17}$$

[2] Loc. cit.

Calculated Gains from Arrays

The flow of power through each unit area due to an advancing electric wave is given by the Poynting vector as

$$s = \frac{c}{4\pi} E \times H \tag{18}$$

where E and H are vectors representing respectively, the electric and magnetic components of the advancing wave.

For free space $|E| = |H|$ so

$$s = \frac{c}{4\pi} E^2. \tag{19}$$

Now the total power radiated through a sphere enclosing an array of sources is

$$P_1 = \int s d\sigma = \frac{c}{4\pi} \int_0^{\pi} \int_0^{2\pi} E_1^2 \sin\theta \, d\phi d\theta. \tag{20}$$

A second system would give

$$P_2 = \frac{c}{4\pi} \int_0^{\pi} \int_0^{2\pi} E_2^2 \sin\theta \, d\phi d\theta. \tag{21}$$

The radiated powers of these two systems might be so adjusted at the source as to give equal fields at any point along a preferred direction. A ratio of these powers, therefore, would be a convenient measure of the relative directional properties of the two arrays. This "test ratio" may conveniently be set up in terms of the equations of the diagrams derived above. In which case

$$T = \frac{\int_0^{\pi} \int_0^{2\pi} r_1^2 \sin\theta \, d\phi d\theta}{\int_0^{\pi} \int_0^{2\pi} r_2^2 \sin\theta \, d\phi d\theta} \tag{22}$$

If we assume all comparisons are to be made with respect to a single linear oscillator the denominator reduces to $8\pi/3$, so

$$T = \frac{3}{8\pi} \int_0^{\pi} \int_0^{2\pi} r_1^2 \sin\theta \, d\phi d\theta. \tag{23}$$

This ratio may conveniently be expressed in decibels. In which case $G = 10 \log_{10} 1/T$ is sometimes called the gain of an array.

If we are interested in the solid array shown in Fig. 21, where $n \cdot N \cdot \mathcal{N}$ linear oscillators, each having respective space and phase separations of $a\lambda$, bT; $A\lambda$, BT; and $\mathcal{A}\lambda$, $\mathcal{B}T$, are arranged progressively along the three principal coördinate axes, this becomes

$$
\begin{aligned}
T = \frac{3}{8\pi}\int_0^{\pi}\int_0^{2\pi} & \frac{\sin^2\left[n\pi(a\cos\phi\sin\theta + b)\right]}{n^2\sin^2\left[\pi(a\cos\phi\sin\theta + b)\right]} \\
& \cdot\frac{\sin^2\left[N\pi(A\sin\phi\sin\theta + B)\right]}{N^2\sin^2\left[\pi(A\sin\phi\sin\theta + B)\right]} \\
& \cdot\frac{\sin^2\left[\mathcal{N}\pi(\mathcal{A}\cos\theta + \mathcal{B})\right]}{\mathcal{N}^2\sin^2\left[\pi(\mathcal{A}\cos\theta + \mathcal{B})\right]} \\
& \cdot\sin^3\theta\, d\phi d\theta.
\end{aligned}
\tag{24}
$$

This integration has been carried out by R. M. Foster who has very kindly placed the results at the writer's disposal. Only the final result is given herewith:

$$
\begin{aligned}
T = \frac{1}{nN\mathcal{N}} &+ \frac{3}{n^2N\mathcal{N}}\sum_{k=1}^{n-1}(n-k)\cdot\cos(2\pi kb)\cdot Q(2\pi ka, 0) \\
&+ \frac{3}{nN^2\mathcal{N}}\sum_{K=1}^{N-1}(N-K)\cdot\cos(2\pi KB)\cdot Q(2\pi KA, 0) \\
&+ \frac{3}{nN\mathcal{N}^2}\sum_{\mathcal{K}=1}^{\eta-1}(\mathcal{N}-\mathcal{K})\cdot\cos(2\pi\mathcal{K}\mathcal{B})\cdot Q(0, 2\pi\mathcal{K}\mathcal{A}) \\
&+ \frac{6}{n^2N^2\mathcal{N}}\sum_{k=1}^{n-1}\sum_{K=1}^{N-1}(n-k)(N-K)\cdot\cos(2\pi KB) \\
&\qquad\cdot\cos(2\pi kb)\cdot Q(2\pi\sqrt{k^2a^2+K^2A^2}, 0) \\
&+ \frac{6}{nN^2\mathcal{N}^2}\sum_{K=1}^{N-1}\sum_{\mathcal{K}=1}^{\eta-1}(N-K)(\mathcal{N}-\mathcal{K})\cdot\cos(2\pi KB) \\
&\qquad\cdot\cos(2\pi\mathcal{K}\mathcal{B})\cdot Q(2\pi KA, 2\pi\mathcal{K}\mathcal{A}) \\
&+ \frac{6}{n^2N\mathcal{N}^2}\sum_{k=1}^{n-1}\sum_{\mathcal{K}=1}^{\eta-1}(n-k)(\mathcal{N}-\mathcal{K})\cdot\cos(2\pi kb) \\
&\qquad\cdot\cos(2\pi\mathcal{K}\mathcal{B})\cdot Q(2\pi ka, 2\pi\mathcal{K}\mathcal{A}) \\
&+ \frac{12}{n^2N^2\mathcal{N}^2}\sum_{k=1}^{n-1}\sum_{K=1}^{N-1}\sum_{\mathcal{K}=1}^{\eta-1}(n-k)(N-K)(\mathcal{N}-\mathcal{K}) \\
&\qquad\cdot\cos(2\pi kb)\cdot\cos(2\pi KB)\cdot\cos(2\pi\mathcal{K}\mathcal{B}) \\
&\qquad\cdot Q(2\pi\sqrt{k^2a^2+K^2A^2}, 2\pi\mathcal{K}\mathcal{A})
\end{aligned}
\tag{25}
$$

Where the function

$$
Q(x, y) = \frac{x^2}{(x^2+y^2)^{3/2}}\sin\left(\sqrt{x^2+y^2}\right) + \frac{x^2-2y^2}{(x^2+y^2)^2}\cos\left(\sqrt{x^2+y^2}\right)
$$

(*Cont.*)

$$-\frac{x^2 - 2y^2}{(x^2 + y^2)^{5/2}} \sin\left(\sqrt{x^2 + y^2}\right) \tag{25a}$$

In particular

$$Q(x, 0) = \frac{\sin x}{x} + \frac{\cos x}{x^2} - \frac{\sin x}{x^3} \tag{25b}$$

and

$$Q(0, x) = -\frac{2 \cos x}{x^2} + \frac{2 \sin x}{x^3} \tag{25c}$$

Special Cases

(1) If we assume $n=2$, $a=\frac{1}{4}$, $b=-\frac{1}{4}$ and $B=\mathcal{B}=0$, the test ratio is given by

$$\begin{aligned} T_1 = \frac{1}{2N\mathcal{N}} &+ \frac{3}{2N^2\mathcal{N}} \sum_{K=1}^{N-1} (N-K) \cdot Q(2\pi KA, 0) \\ &+ \frac{3}{2N\mathcal{N}^2} \sum_{\mathcal{K}=1}^{\eta-1} (\mathcal{N}-\mathcal{K}) \cdot Q(0, 2\pi\mathcal{K}\mathcal{A}) \\ &+ \frac{3}{N^2\mathcal{N}^2} \sum_{K=1}^{N-1} \sum_{\mathcal{K}=1}^{\eta-1} (N-K)(\mathcal{N}-\mathcal{K}) \\ &\cdot Q(2\pi K\mathcal{A}, 2\pi\mathcal{K}\mathcal{A}) \end{aligned} \tag{26}$$

This, like equation (13), corresponds to the practical case of transmission from an antenna curtain and reflector each made up of N vertical columns of $\mathcal{N}$ elements, all driven in the same phase.

(2) If we assume that no stacking is involved, then $\mathcal{N}=1$ and we have for the test ratio for N couplets

$$\begin{aligned} T_2 &= \frac{1}{2N} + \frac{3}{2N^2} \sum_{K=1}^{N-1} (N-K) \cdot Q(2\pi KA, 0) \\ &= \frac{1}{2N} + \frac{3}{2N^2} \sum_{K=1}^{N-1} (N-K) \cdot \left[\frac{\sin 2\pi KA}{2\pi KA} \right. \\ &\qquad \left. + \frac{\cos 2\pi KA}{(2\pi KA)^2} - \frac{\sin 2\pi KA}{(2\pi KA)^3} \right]. \end{aligned} \tag{27}$$

This equation was used in the calculation of the data given in Figs. 6, 7, and 8.

(3) If we wish to apply equation (25) to the case of a single array of N linear oscillators driven in phase we have $n = \mathcal{N} = 1$ and $B=0$, so

$$T_3 = \frac{1}{N} + \frac{3}{N^2} \sum_{K=1}^{N-1} (N - K) \cdot Q(2\pi KA, 0) \tag{28}$$

which differs from equation (27) by a factor of two. This indicates that an array of N equiphased linear couplets gives twice the field in the preferred direction as received from N equiphased linear elements radiating the same power.

(4) Applying equation (25) to the extremely simple case of one couplet, $n=2$, $a=\frac{1}{4}$, $b=-\frac{1}{4}$ and $N=\mathcal{N}=1$ and

$$T_4 = \tfrac{1}{2}. \tag{29}$$

(5) We may calculate the test ratio for a single stack of linear couplets (earth effects not considered) by placing $N=1$, $n=2$, $a=\frac{1}{4}$, $b=-\frac{1}{4}$, and $\mathcal{B}=0$ and get

$$\begin{aligned} T_5 &= \frac{1}{2\mathcal{N}} + \frac{3}{2\mathcal{N}^2} \sum_{\mathcal{K}=1}^{\eta-1} (\mathcal{N}-\mathcal{K}) \cdot Q(0, 2\pi\mathcal{K}\mathcal{A}) \\ &= \frac{1}{2\mathcal{N}} - \frac{3}{\mathcal{N}^2} \sum_{\mathcal{K}=1}^{\eta-1} (\mathcal{N}-\mathcal{K}) \left[\frac{\cos(2\pi\mathcal{K}\mathcal{A})}{(2\pi\mathcal{K}\mathcal{A})^2} - \frac{\sin(2\pi\mathcal{K}\mathcal{A})}{(2\pi\mathcal{K}\mathcal{A})^3} \right]. \end{aligned} \tag{30}$$

This equation was used in calculating the data given in Figs 18 and 19.

(6) The test ratio for the case of the rectangular array of nN elements discussed in connection with Fig. 16 may be calculated by placing $\mathcal{N}=1$, $a=\frac{1}{4}$, $b=-\frac{1}{4}$ and $B=0$. In which case

$$\begin{aligned} T_6 = \frac{1}{nN} &+ \frac{3}{nN^2} \sum_{K=1}^{N-1} (N - K) \cdot Q(2\pi KA, 0) \\ &+ \frac{3}{Nn^2} \sum_{k=1}^{n-1} (n - k) \cdot \cos \frac{(k\pi)}{2} \cdot Q\left(\frac{k\pi}{2}, 0\right) \\ &+ \frac{6}{n^2N^2} \sum_{K=1}^{N-1} \sum_{k=1}^{n-1} (n - k)(N - K) \cdot \cos\left(\frac{k\pi}{2}\right) \\ &\cdot Q\left(2\pi\sqrt{\frac{k^2}{16} + K^2A^2}, 0\right). \end{aligned} \tag{31}$$

Areas of Directional Diagrams

In general, the areas of directional diagrams may be calculated from their equations by the usual integration methods. The special case of N couplets in horizontal array, such as used rather generally in practice and shown in Fig. 5 above, is of sufficient importance to be given here. The area of the diagram in the (XY) plane is

$$S = \frac{1}{N^2}\left[\frac{N}{2} + \sum_{K=1}^{N-1}(N - K)\cdot J_0(2\pi KA)\cdot\cos 2\pi KB\right]. \qquad (32)$$

This equation was used in calculating the data given in Fig. 5.

The area of diagrams in the horizontal plane due to a single array of N oscillators is given by the equation:

$$S = \frac{2}{N^2}\left[\frac{N}{2} + \sum_{K=1}^{N-1}(N - K)\cdot J_0(2\pi KA)\cdot\cos 2\pi KB\right]^*. \qquad (33)$$

This differs from equation (32) by a factor of two and indicates that regardless of whether the gain is reckoned by an integration over a unit sphere or in terms of the area of the horizontal diagram the effect of the reflector is to double the radiated field in the preferred direction.

Placing $N=1$ in equation (32)

$$S = \tfrac{1}{2}. \qquad (34)$$

This is analogous to equation (29) above.

Arrays of Arrays

Each element of a generalized linear array, such as shown in Fig. 21‘ may be replaced by a generalized array, thereby producing an array of arrays.[9] It may be shown that the resultant is given by an array factor, representing the characteristics of individual arrays, times other factors representing the relative position of the individual arrays in the array of arrays. A derivation analogous to that beginning on page 22 results in the equation

$$R = r\cdot\frac{\sin n'\pi(a'\sin\phi + b')}{n'\sin\pi(a'\sin\phi + b')}\cdot\frac{\sin N'\pi(A'\sin\phi + B')}{N'\sin\pi(A'\sin\phi + B')}\,\frac{\sin \mathcal{N}'\pi(\mathcal{A}'\sin\phi + \mathcal{B}')}{\mathcal{N}'\sin\pi(\mathcal{A}'\sin\phi + \mathcal{B}')} \qquad (35)$$

where $a'\lambda$, $A'\lambda$ and $\mathcal{A}'\lambda$ are the coordinate spacings between arrays and $b'T$, $B'T$, and $\mathcal{B}'T$ are the corresponding phase intervals, and r represents the characteristics of one of the individual arrays. If each array is of the type shown in Fig. 5, r is given by equation (14) above.
Placing $n'=\mathcal{N}'=1$ and $N'=2$
also $n=2$ and $B=0$, the above equation reduces to

$$R = \frac{\sin N'\pi(A'\sin\phi + B')}{N'\sin\pi(A'\sin\phi + B')}\cdot\frac{\sin N\pi(A\sin\phi)}{N\sin\pi(A\sin\phi)}\cos\frac{\pi}{4}(1 - \cos\phi) \qquad (36)$$

which is that made use of in calculating the diagrams in Figs. 14 and 15.

* R. M. Foster, "Directive diagrams of antenna arrays," *Bell Sys. Tech. Jour.*, **5**, 307; 1926.

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RADIO ELECTRIC CLOCK SYSTEM*

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Summary—*This paper describes briefly a clock system which employs radio time signals from a government station to correct automatically an electric clock system.*

As interference is the usual limitation of a system of this sort, special emphasis has been placed upon the pulse amplifier, by means of which pulses of a periodic character are amplified with an extremely high selectivity against interference. The mathematical theory of this amplifier is developed in detail, and response curves for several stages are drawn.

Introduction

THIS PAPER is intended to present the main features of a clock system which employs radio time signals for automatic correction. While the problems are essentially of a special character many of them may be of general interest.

The broad ideas and many of the elements of the system have been the work of T. S. Casner, Chief Engineer of the Radio Electric Clock Corporation; the radio-receiving arrangement is due to L. A. Hazeltine, formerly Professor of Electrical Engineering at Stevens Institute of Technology, while the writers have been associated in its development.

Description of Operation

The elements of the system consist of a master clock, secondary clocks, a radio receiver, and a magnetic selector. The operation is as follows: Approximately at 11:55 A.M. each day the master clock closes a contact which turns on the filament circuit of the receiver, putting it into readiness to receive the time signals. At 11:55 A. M. the Government Station NAA commences to send time signals. These signals are sent on a radio-frequency carrier of 112 kc, modulated at 500 cycles, and consists of pulses of 0.35-sec. length once per sec. The 29th pulse and the last 5 pulses of each minute are omitted until the last minute of the hour, when the 29th and the last 10 pulses are omitted. The signals end at 12 M. with a pulse of one second duration.

The function of the radio receiver is to amplify the incoming radio-frequency waves, detect them to produce one-second pulses (direct-

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current), and then amplify these pulses to a magnitude suitable for operating the magnetic selector.

The selector is a magnetic device which is so constructed as to bring into alignment two contact pieces during the 5th second of the silent interval at the end of each minute. Thus, normally, the selector contact will be aligned during the 60th second of each of the first four minutes of signal broadcast. At the beginning of the first second of the new minute, another pulse will come through, which will cause the selector to push the two aligned contact pieces together, thereby closing a circuit which actuates a magnet, setting the master clock to the exact second. In general the error of the master clock seldom exceeds a few seconds. The maximum possible correction is ± 25 seconds.

The secondary clocks are of a magnetic type and are actuated from the master clock once each minute. As they do not form an essential part of the radio system they will be omitted from further discussion.

The Radio Receiver

General Features

The radio receiver presents many novel features due mostly to the severe requirements which must be met in order to insure successful operation. These are listed below as follows:

1. Receiver must have high selectivity against other signals and especially static.
2. Receiver must be capable of being operated perhaps several hundred feet from the antenna.
3. Receiver must be capable of operating on ordinary commercial power circuits.
4. Receiver and associated selecting apparatus must be compact.
5. Receiver should include a source of signals for test purposes.

Requirement (1) is probably the most difficult of the five, from the technical standpoint. Its fulfillment must be inherent in the nature of the apparatus if it is to be at all successful.

Interference in the nature of code and static will superimpose irregular pulses upon the Arlington pulses, which will result in irregular action of the selector and may prevent the time correction from being made if the interfering signals persist in the silent interval.

The most natural solution of this problem apparently would be to employ a sharply tuned radio-frequency amplifier which would give good selection at the wave frequency. Upon detection this amplifier would yield the 500-cycle modulation of the Arlington signal which could be passed through an audio amplifier tuned to 500 cycles. This amplifier would be followed by a second detector producing pulses (i.e.,

the envelope of the 500-cycle wave trains) suitable for operating the selector. An alternative solution would be the employment of heterodyne reception, with audio-frequency tuning to the beat note.

The disadvantage of such solutions is that static and other interference, after detection, leave components whose frequencies lie within a relatively wide audio band. The effectiveness of these components will be determined by the width of the resonance band of the audio amplifier, which usually is broad.

In the system to be described the received signal is first amplified at the wave frequency in a sharply tuned amplifier, then detected, and finally amplified in a special amplifier effective for the pulse frequency only. The width of the resonance band of the pulse amplifier, (Fig. 11), is relatively narrow compared with that feasible in an audio amplifier. The result is the highest attainable selectivity against static with a considerable simplification in apparatus. The pulse-frequency amplifier is probably the most novel part of the receiver and represents a device, suitable not only for amplifying Arlington pulses, but all manner of pulses which are used in telegraphy and automatic control devices.[1]

The remaining four requirements enumerated above represent desirable features intimately associated with commercial application.

In a great many cases where the system is installed in a large office building, it is imperative that the receiver be placed a considerable distance from the antenna. In such cases the antenna is terminated in a specially tuned circuit called the "antenna tuner." From the antenna tuner the radio-frequency signals are transmitted to the receiver through an ordinary two-wire armored cable or metal conduit. The antenna tuner serves also to adjust the impedance of the antenna circuit to that of the armored-cable transmission line. Likewise, an impedance-adjusting device is provided to connect the cable to the receiver.

Items (3), (4), and (5) are so closely associated with the mechanical construction that they will be discussed in the next section dealing with that subject.

General Construction

A general idea of the arrangement and function of the various pieces of apparatus can be obtained from Fig. 1. The antenna tuner serves to tune the antenna circuit, and is connected to the antenna lead-in directly where it enters the building. From the antenna tuner the

[1] For a more complete discussion of this subject, see U. S. Letters Patent No. 1,656,888 issued to L. A. Hazeltine.

signals are transmitted over a shielded two-wire line to the radio receiver. Here the signals are converted to direct-current pulses, which are carried over a two-wire line to the selector in the master clock. The selector controls an electromagnet which sets the master clock at the proper time. Power for operating the receiver is obtained either from batteries or the power boxes shown.

The radio receiver is of the neutrodyne type having two stages of radio-frequency amplification, a detector, two stages of reflexed pulse-frequency amplification, and one ordinary stage of pulse-frequency amplification. The saving due to reflexing is not only the cost and space required by the tubes but also the radio-frequency filter apparatus which otherwise would be necessary. The circuit diagram, shown in Fig. 2, shows the complete wiring scheme.

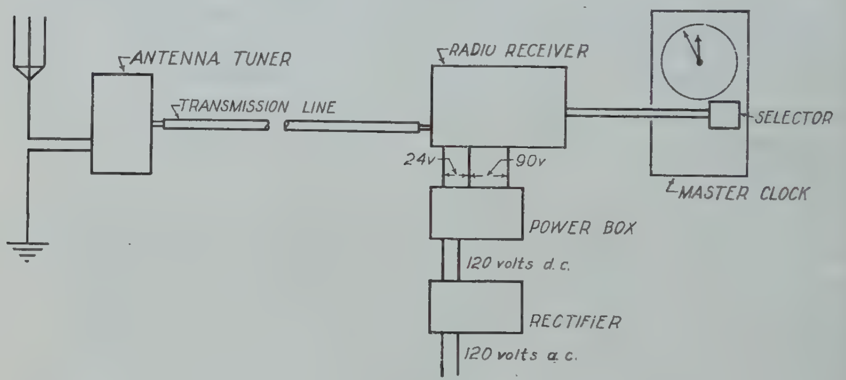
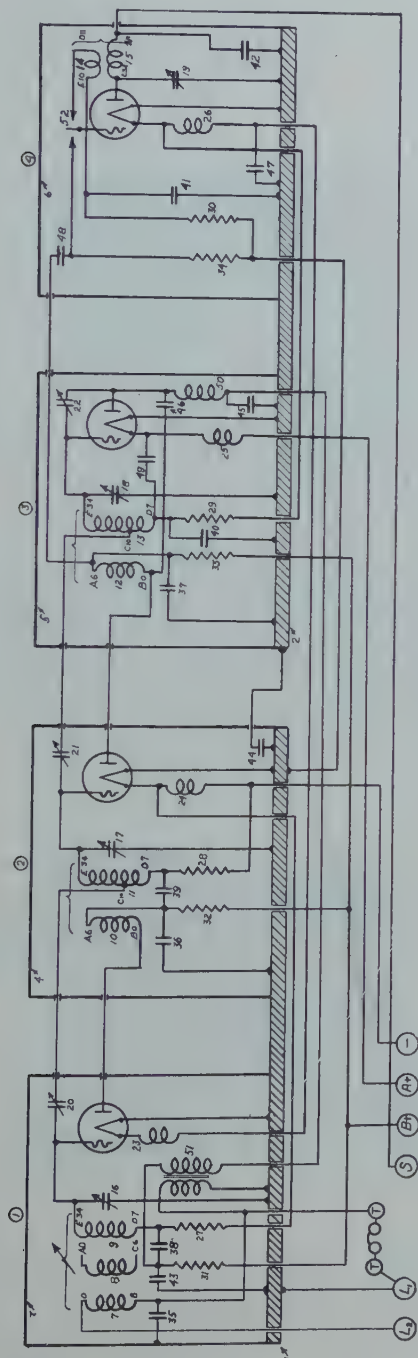


Fig. 1

Advantage is taken of the output tube, shown in can 4 of Fig. 2, to serve also as a signal generator for test purposes. It is, therefore, provided with a tuned circuit so that it can oscillate at a frequency of 112 kc per sec. when desired. Each radio-frequency unit is thoroughly shielded by enclosing it in a can made of 1/32 in. copper screwed down to a 3/32 in. copper base plate.

The tube filaments and the filament chokes provided for each tube are connected in series. This forms a low-current filament circuit suitable for operation from an a-c circuit through a rectifier. It also provides a convenient means of obtaining various bias voltages. The base plate is divided into two parts; one forming the common terminal of the first and second tube filaments, and the other the common terminal of the third and fourth tube filaments. Instead of mounting these two base plates next to each other, so that all four units would be side by side, the third and fourth units are turned upside down and mounted under the second and first units respectively. This arrange-



Circuit Diagram of Radio Electric Clock Receiver.
Fig. 2

ment greatly shortens the wiring between the various units and consequently decreases the chance of the stray pickup. The resulting receiver shown in Fig. 3 is very small, being only 11 in. wide, 11 in. high, and 9 in. deep.

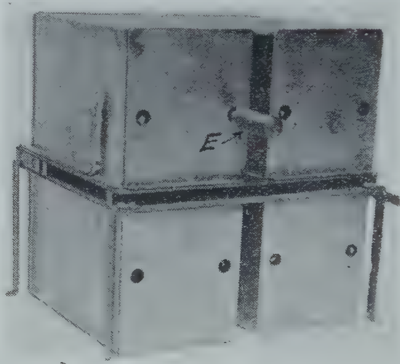


Fig. 3

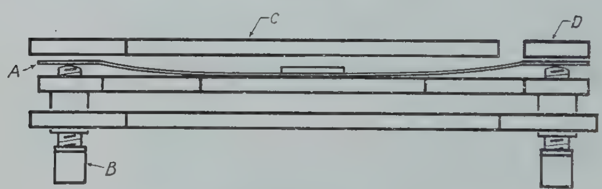


Fig. 4

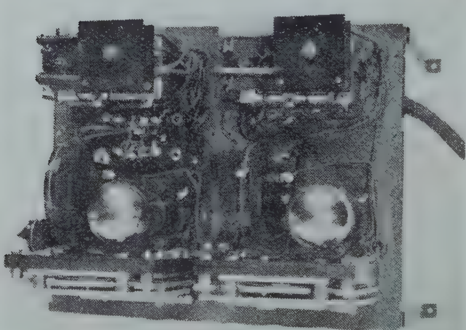


Fig. 5

The tuning units in this receiver are not of the conventional type. The desire for efficiency at the comparatively low frequency leads to the pancake type of coil. (See Figs. 5 and 6.) They are wound with litzendraht to cut down eddy-current losses. The layers are spaced

from each other with thick paper to decrease the dielectric and eddy-current losses. To decrease eddy-current losses in the shields, care has been taken to mount the coils as far from them as possible. To prevent stray coupling the chokes and tuning coils in any one unit are mounted at right angles. Figs. 5 and 6 show the interior of the first two units. The tuning condensers, (Figs. 4, 5, and 6), are composed of three aluminum plates so fixed with respect to each other that the capacitance between the inner one and the outer pair is just a little less than that required for tuning.

Referring to Fig. 4: Flexible plate *A* operated against plate *C* by screw *B* forms a padding condenser which provides a fine tuning adjust-

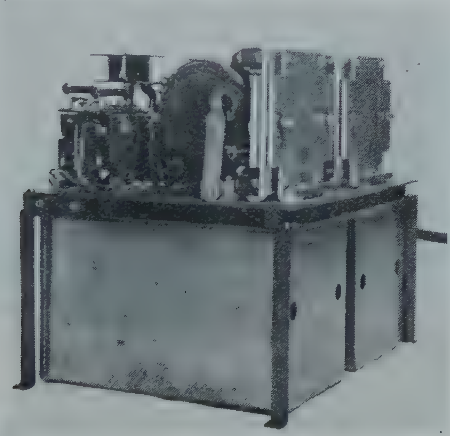


Fig. 6

ment. The other end of plate *A* and small plate *D* provides a variable neutralizing condenser. The adjusting screws for tuning and neutralizing are operated by means of a bakelite wrench *E* which is inserted in a hole in each shield. (See Fig. 3).

The input coil 7, shown in Fig. 2, serves to couple the first tuned circuit to the transmission line. It is mounted on an arm so that the coupling between it and its secondary coil 9, may be varied from a maximum value to zero. When in this latter position, no audible signal should come through the phones either from the antenna or from the test oscillator. This provides a means of determining the presence of stray pickup in the receiver.

The two daily five-minute periods during which Arlington sends time signals are quite insufficient for testing purposes. This difficulty has been overcome as explained before by making the output tube of the pulse amplifier both an oscillator and an amplifier. By turning a small

switch, 52, (Figs. 2 and 6), the operator can make the tube produce a self-modulated signal at Arlington's wavelength which will start a pulse through the radio-frequency end. Before the pulse reaches the last tube of the receiver from which it originated, this tube may be switched back to its amplifier circuit, and it will thus amplify at pulse frequency the modulated radio-frequency pulse which it produced.[2] The pulse so produced will actuate the selector in the normal manner. By slowly oscillating the switch controlling the last tube the operator can simulate the normal Arlington signal, thereby checking the tuning of the radio-frequency end of the receiver and also the selector mechanism.

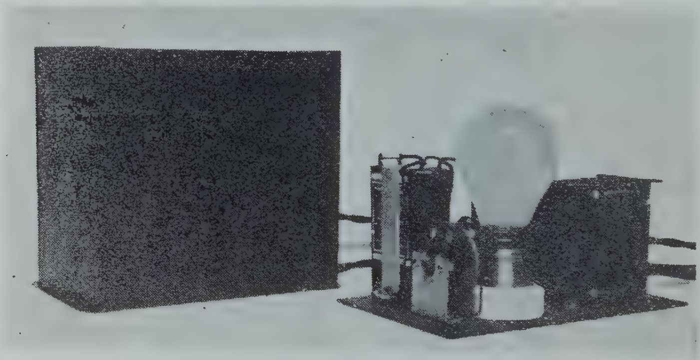


Fig. 7

The receiver has been designed to operate directly from a 24-volt "A" battery and a 90-volt "B" battery. These same voltages can be obtained from a 120-volt direct- or alternating-current power line by using a suitable power box.

The power box for use on a 120-volt direct-current line, as shown in Fig. 7. consists of a filter system, followed by a group of resistances to provide the proper voltages for the receiver. The filter consists of an iron-core choke having two low resistance windings, one in each side of the line, followed by a condenser across the line. The negative side of the line is grounded through a condenser. The purpose of this filter is to prevent radio-frequency and audio-frequency disturbances from entering the receiver through the power lines. A part of the series resistance that reduces the supply to 24 volts for the filaments is a Mazda lamp. This provides a visual means of determining whether or not the set is turned on.

[2] As will be explained later, the pulse suffers a time delay in passing through the pulse amplifier which makes this possible.

The power box is adapted to an alternating-current supply by interposing a dry rectifier of the copper oxide type. The rectified output goes to a regular direct-current power box, to which has been added another stage of filtering. This is indicated in the schematic drawing of Fig. 1. There is a considerable ripple in the output wave. Although it can be heard clearly in the phones in the detector circuit, it has very little effect on the output pulse. The reason for this can be seen by referring to Fig. 11, which shows that the gain of the pulse amplifier at 60 cycles is negative.

Radio-Frequency Amplifier

The characteristic requirement of this amplifier is that it must tune sharply to a single frequency. This requirement arises primarily not so much from the desire to eliminate possible interfering stations as from the desire to cut down the received energy of static interference. As static has components of all frequencies, the received energy due to static is almost proportional to the width of the resonance band. This is the primary reason for requiring extreme selectivity.

The selectivity of a circuit is determined by its natural power factor. Considering only the tuned circuit, its power factor can be considered equal to that of the coil plus that of the condenser and grid conductance of the following tube. The power factor of a coil at a given frequency is a function of its bulk, which in turn is usually determined by either space or cost limitations. Thus, for high selectivity, the additional power factor due to the insulating supports and grid conductance, which is equal to $g/\omega C$ should be kept down to a small part of the coil power factor. As g is a constant ωC should be large.

As high amplification and selectivity both demand opposite things a compromise must be made with the net result that it is impractical to use maximum possible amplification per stage.[3]

The actual amplification per stage, however, is about thirty, a figure about three times as high as the average broadcast receiver using 201-A tubes. The reason for this is simply the lower frequency which allows a greater step-up ratio in the radio-frequency transformer.

In order to avoid energy feedback between stages due to the high gain, the individual stages are carefully neutralized and shielded. In the radio-frequency amplifier stages neutralization is effected in the ordinary manner, using the condensers 20 and 21 (shown in Fig. 2) connected from the grid of their respective tubes to a tap on the secondary of the radio-frequency transformers connected in their plate circuits. In the detector, however, instead of neutralizing the regenera-

[3] L. A. Hazeltine, "Discussion on the shielded neutrodyne receiver by Dreyer and Manson." Proc. I. R. E., **14**, 395; June, 1926.

tive effect of the tube grid-plate capacity, it becomes necessary to neutralize the anti-regenerative effect, due to the detector plate circuit having a capacitive reactance. This anti-regenerative effect is very undesirable as it decreases the selectivity. It is eliminated by grid circuit neutralization, using the condenser 46 connected from the detector plate to the primary of the detector radio-frequency transformer. As it is inconvenient to vary condenser 46, this condenser is made fixed, of a size slightly larger than necessary, and a variable condenser 22 is shunted around the tube grid-plate capacity. By adjusting condenser 22, the detector circuit may be neutralized or, if it is desirable, made slightly regenerative.

Another aid in preventing energy feed back, which is somewhat novel, is the reflexed pulse amplifier, which provides a filter consisting of a 4-μf condenser and a 50,000-ohm resistor in the plate circuit of each radio-frequency tube. This filter combination is effective in returning the radio-frequency plate current directly to its filament after it has passed through the primary of the radio-frequency transformer.

Plate detection is used in the detector circuit, to avoid the high grid conductance that would occur with a positive grid necessary when using grid detection. For this purpose a 2-megohm resistance is connected in series in the detector plate circuit, which reduces the effective voltage on the detector plate to a value low enough to give good rectification with the $4\frac{1}{2}$-volt bias used.

In order to obtain audible signals (the 500-cycle modulation of Arlington,) which are desirable for tuning purposes, a head set must be connected in the detector circuit. For this purpose a step-down transformer, 51, shown in Fig. 2, must be used in series with the detector plate circuit to adjust the impedance of the ordinary head set to the high impedance of the detector circuit.

Antenna Tuner

The antenna tuner as shown in Fig. 8 is simply a device which tunes the antenna circuit and thus contributes to the over-all selectivity of the receiver. It is made as a separate shielded unit so that it may be placed at the termination of the antenna in the building away from the receiver if necessary.

When the antenna tuner and receiver are separated it is necessary to connect them by a wire line of some kind that will pick up as little noise as possible. The most satisfactory line which has been used for this purpose is a BX armored cable or iron conduit, such as is ordinarily used for lighting circuits.

Such a line will naturally have a low characteristic impedance,

owing to its relatively low inductance and high capacitance. Maximum energy transfer along such a circuit occurs when it works out of and into its characteristic impedance, which calls for a considerable step-down of voltage at the sending end, and a corresponding voltage step-up at the other end.

The actual voltage ratio in the antenna tuner or the radio-frequency transformer of the first stage at maximum coupling is approximately 80 to 1, which gives an impedance ratio of 6400 to 1. Loosening the coupling at either end provides a means for adjusting the intensity of the signal to that required for operating the selector with only a small margin, thereby minimizing interference.

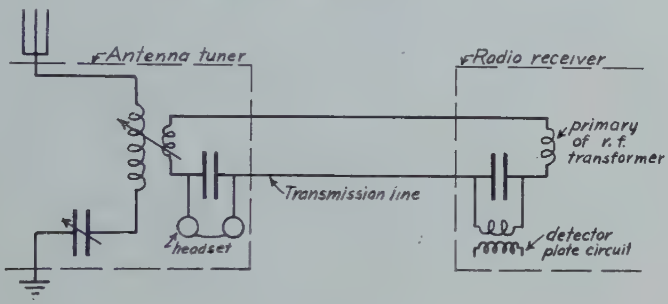


Fig. 8

A particularly interesting feature of the transmission line is that it can be used for audio frequencies as well as radio frequencies. This is particularly useful when tuning the antenna tuner as it allows the operator to hear the signals even though the receiver is not situated at the antenna. As is illustrated in Fig. 8, this is accomplished by connecting the audio-frequency transformer of the detector-plate circuit in series with the transmission line at the receiver end, and a telephone head set in series with it at the antenna-tuner end. Both the audio-transformer secondary and the head set are shunted by a 0.1-μf condenser to offer a low impedance path for the radio-frequency antenna current.

The Pulse Frequency Amplifier

As was mentioned earlier, the pulse-frequency amplifier is specially designed to amplify only a narrow band of frequencies; in this particular case the frequency for maximum amplification is about one cycle per second. Thus all noise frequencies and static pulses having a fundamental frequency very different from one cycle per second will be highly attenuated.

What has just been said applies to that method of analysis which considers the periodic rectangular pulse as made up of a harmonic

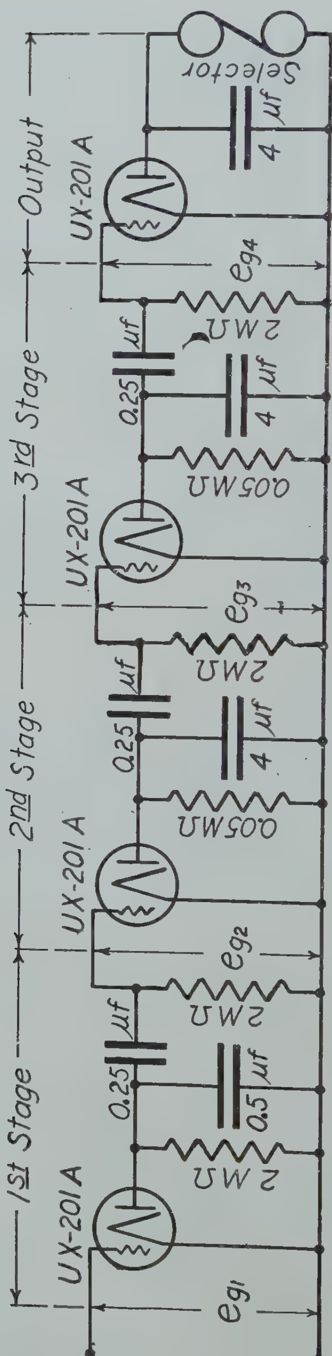

Output
3rd Stage
2nd Stage
1st Stage
UX-201 A
UX-201 A
UX-201 A
UX-201 A
Selector
4 μf
0.25 μf
2 MΩ
4 μf
0.05 MΩ
0.25 μf
2 MΩ
4 μf
0.05 MΩ
0.25 μf
2 MΩ
0.5 μf
2 MΩ
e_{g4}
e_{g3}
e_{g2}
e_{g1}

Fig. 9

series. A more accurate idea of the response of the amplifier can be obtained if the analysis is based on the transient response to a suddenly applied voltage. Both methods will be discussed.

The following circuit diagram, Fig. 9, is that for a three-stage pulse amplifier and the output tube. Battery wires, etc., are not shown.

A simplified circuit for one stage is shown in Fig. 10.

$\boldsymbol{E}_{g1}$ is the equivalent pulse voltage which may be considered to be placed upon the grid of the detector. It will produce a voltage $\mu \boldsymbol{E}_{g1}$ in the detector plate circuit. The simplified circuit of Fig. 10 replaces the vacuum tube by a generator of $\mu \boldsymbol{E}_{g1}$ volts in series with a constant plate conductance of g_p *mhos.*[4]

In this circuit, g is the conductance of the coupling resistance; C is the capacitance of the shunt condenser which cuts off the higher frequencies; C_1 that of the grid coupling condenser and g^1 is the grid-leak conductance.

Let us first find the steady-state voltage amplification per stage for a sinusoidal impressed voltage; $\mu \boldsymbol{E}_{g1}$ then represents the r.m.s. value of the fundamental pulse component of the signal, or random interference, considered to be impressed in the plate circuit of the detector.

We may then write for the ratio of the voltage across the capacitance to that impressed in the plate circuit

$$\frac{\boldsymbol{E}_c}{\mu \boldsymbol{E}_{g1}} = \frac{\dfrac{1}{g + j\omega C + \dfrac{1}{\dfrac{1}{j\omega C_1} + \dfrac{1}{g'}}}}{\dfrac{1}{g_p} + \dfrac{1}{\dfrac{1}{g + j\omega C + \dfrac{1}{\dfrac{1}{j\omega C_1} + \dfrac{1}{g'}}}}} \tag{1}$$

The ratio of output voltage $\boldsymbol{E}_{g2}$ to $\boldsymbol{E}_c$ will be

$$\frac{\boldsymbol{E}_{g2}}{\boldsymbol{E}_c} = \frac{\dfrac{1}{g'}}{\dfrac{1}{j\omega C_1} + \dfrac{1}{g'}} \tag{2}$$

[4] For the purpose in hand it is sufficiently accurate to treat the tube conductances as constant.

If (1) and (2) are multiplied together and simplified, the amplification per stage will be

$$\frac{E_{g2}}{E_{g1}} = \mu g_p \frac{j\omega C_1}{(j\omega)^2 C_1 C + j\omega(C_1 g + C_1 g' + C g' + C_1 g_p) + g'(g_p + g)} \qquad (3)$$

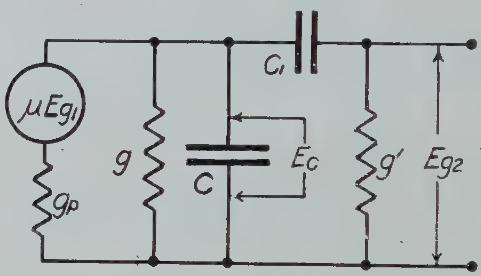


Fig. 10

If we now group (3) into its real and imaginary parts, the magnitude of the amplification after simplification may be written

$$\frac{E_{g2}}{E_{g1}} = \frac{\mu g_p}{\left(g_p + g + g'\left[1 + \frac{C}{C_1}\right]\right)\left[1 + \left(\frac{\frac{g'(q_p+g)}{C_1\omega} - C\omega}{g_p+g+g'\left[1+\frac{C}{C_1}\right]}\right)^2\right]^{1/2}} \qquad (4)$$

Letting $g^1(g+g_p)/C_1C = \omega_0^2$; where ω_0 represents the frequency giving maximum amplification, and substituting in (4) we have after simplification

$$\frac{E_{g2}}{E_{g1}} = \frac{\mu g_p}{\left(g_p+g+g'\left[1+\frac{C}{C_1}\right]\right)\left[1 + \left(\frac{\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}}{\frac{g_p+g+g'\left[1+\frac{C}{C_1}\right]}{\omega_0 C}}\right)^2\right]^{1/2}} \qquad (5)$$

Now let $\qquad x = \log_\epsilon \frac{\omega}{\omega_0}$

and

$$p = \frac{g_p + g + g'\left[1 + \frac{C}{C_1}\right]}{\omega_0 C}$$

Where p represents the ratio of the effective conductance to admittance of the circuit at the frequency of maximum amplification; substituting these into (5) we have

$$\frac{E_{g2}}{E_{g1}} = \frac{\mu\gamma_p}{\left(g_p + g + g'\left[1 + \frac{C}{C_1}\right)\left\{1 + \left(\frac{\epsilon^{-x} - \epsilon^{x}}{p}\right)^2\right\}^{1/2}} \quad (6)$$

The amplification per stage in decibels will then be

$$20 \log_{10} \frac{\mu\gamma_p}{g_p + g + g'\left[1 + \frac{C}{C_1}\right]} - 10 \log_{10}\left(1 + \frac{4 \sinh^2 x}{p^2}\right) db. \quad (7)$$

For n stages the amplification will be

$$20n \log_{10} \frac{\mu\gamma_p}{g_p + g + g'\left[1 + \frac{C}{C_1}\right]} - 10n \log_{10}\left(1 + \frac{4 \sinh^2 x}{p^2}\right) db. \quad (8)$$

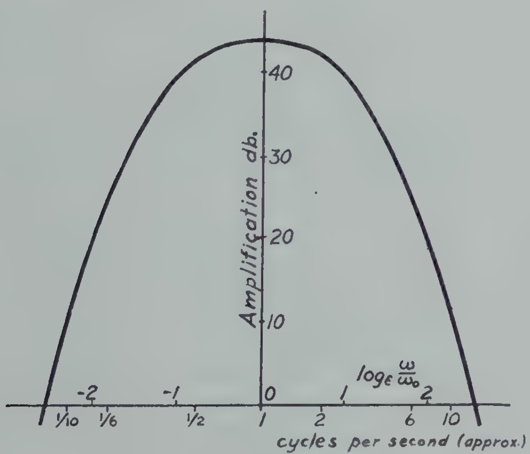


Fig. 11

The amplification characteristic of the three-stage amplifier as a function of $x = \log_\epsilon \omega/\omega_0$ is shown plotted in Fig. 11. It is symmetrical about $x = 0$, as shown by (8). A scale of frequency in cycles per second is also given.

Using the constants shown in diagram of Fig. 9 the maximum amplification at ω_0 for three stages is approximately

$$20n \log_{10} \frac{\mu g_p}{g_p + g + g'\left[1 + \dfrac{C}{C_1}\right]} = 44db$$

$$\omega_0 = \left(\frac{g'(g + g_p)}{C_1 C}\right)^{1/2} = 6 \text{ radians per second}$$

$$p = \frac{g_p + g + g'\left[1 + \dfrac{C}{C_1}\right]}{\omega_0 C} = 3$$

Whereas the actual constants for the detector stage and the two following stages are quite different, they are so chosen that the values of ω_0 and p listed above are approximately the same for each stage, which justifies calculating the amplification of the three as a unit.

The maximum amplification occurs at one cycle per sec. which is the Arlingtonpulse frequency. Frequencies below one cycle per sec. are rapidly attenuated; this is desirable as these frequencies represent power-line fluctuations and similar disturbances which are common. On a percentage basis frequencies above one cycle per sec. are also rapidly attentuated, thus 2 cycles per sec. is amplified 95 per cent as much as the maximum, 6 cycles per sec. about 55 per cent as much, while at approximately 16 cycles per sec. the gain is zero. This small spread of the amplification band is rather desirable as it allows the harmonics of the rectangular pulse from Arlington to be amplified, which will tend to make the output far more like the input wave then it otherwise would be. At 60 cycles per sec. the gain will be very negative so that no disturbance need be expected when using a 60-cycle power supply which is poorly filtered. This accounts for the fact that very little filtering apparatus is necessary in the power-supply unit.

The clearest idea of the exact action of the amplifier can be obtained from its transient characteristics. The simplest transient characteristic to consider is the voltage response to a suddenly applied and continuously maintained voltage. This particular problem lends itself admirably to solution by means of operational calculus.[5] If desired it also can be solved by the more classical methods usually applied to differential equations.

In terms of operational calculus this applied voltage may be written as $e_{g1}1$ where the 1 signifies a unit voltage suddenly applied at time zero and maintained at unit value thereafter. To obtain the response to the Arlington pulse which is maintained only for 0.35 sec. it is simply

[5] See any of the textbooks now available, such as by V. Bush, "Operational Circuit Analysis," etc.

necessary to superimpose on the response to $e_{g1}1$ the response due to $-e_{g1}1$ applied at the time 0.35 sec.

The response due to $e_{g1}1$ may be obtained by replacing $(j\omega)$ in (3) by the symbol $p = d/dt$ and evaluating the resultant operational expression, as written below

$$e_{g2} = \frac{\mu g_p C_1 p}{C_1 C p^2 + (C_1 g + C g' + C_1 g' + C_1 g_p)p + g'(g_p + g)} e_{g1}1 \quad (9)$$

$$= \frac{\mu g_p}{C} \frac{p}{p^2 + \frac{C_1 g + C g' + C_1 g' + C_1 g_p}{C_1 C} p + \frac{g'(g_p + g)}{C_1 C}} e_{g1}1 \quad (10)$$

If we let

$$2b = \frac{C_1 g + C g' + C_1 g' + C_1 g_p}{C_1 C} = \frac{g'}{C_1} + \frac{g' + g + g_p}{C}$$

and

$$a = \frac{g'(g_p + g)}{C_1 C}$$

then e_{g2} may be written

$$e_{g2} = \frac{\mu g_p}{C} \frac{p}{p^2 + 2bp + a} e_{g1}1. \quad (11)$$

If the determinantal equation is set equal to zero, that is

$$p^2 + 2bp + a = 0$$

it will yield two roots, as follows

$$\lambda_1 = -b - (b^2 - a)^{1/2}$$

$$\lambda_2 = -b + (b^2 - a)^{1/2}$$

Equation (11) may now be written

$$e_{g2} \doteq \frac{\mu g_p}{C} \frac{p}{(p - \lambda_1)(p - \lambda_2)} e_{g1}1. \quad (12)$$

It is convenient to break the operator of (12) into partial fractions yielding

$$e_{g2} = \frac{\mu g_p}{C} \left(\frac{-\lambda_1}{\lambda_2 - \lambda_1} \frac{1}{p - \lambda_1} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \frac{1}{p - \lambda_2} \right) e_{g1}1. \quad (31)$$

Evaluating the operational expression we obtain[6]

$$e_{g2} = e_{g1}\frac{\mu g_p}{C}\frac{1}{\lambda_2 - \lambda_1}(\epsilon^{\lambda_2 t} - \epsilon^{\lambda_1 t})1. \tag{14}$$

But $\lambda_2 - \lambda_1 = 2(b^2 - a)^{1/2}$ which we shall call h and for convenience we shall call $-\mu g_p/C = f$

then
$$e_{g2} = -e_{g1}\frac{f}{h}(\epsilon^{\lambda_1 t} - \epsilon^{\lambda_2 t})1 \tag{15}$$

which is the response of the first stage.

As λ_2 is smaller than λ_1, the final result will be negative, which makes it convenient to consider the initial minus sign as showing the direction of the output pulse voltage e_{g2} for a positive input pulse e_{g1}.

In considering the second stage, which may be assumed identical with the first,[7] we may use (13) again, and obtain

$$e_{g3} = \frac{\mu g_p}{C}\left(\frac{-\lambda_1}{\lambda_2 - \lambda_1}\frac{1}{p - \lambda_1} + \frac{\lambda_2}{\lambda_2 - \lambda_1}\frac{1}{p - \lambda_2}\right)e_{g2}1. \tag{16}$$

Substituting in the value of e_{g2} from (15) we get

$$e_{g3} = \frac{\mu g_p}{C}\left(\frac{-\lambda_1}{\lambda_2 - \lambda_1}\frac{1}{p - \lambda_1} + \frac{\lambda_2}{\lambda_2 - \lambda_1}\frac{1}{p - \lambda_2}\right)\left(-e_{g1}\frac{f}{h}\right)\left((\epsilon^{\lambda_1 t} - \epsilon^{\lambda_2 t}\right)1 \tag{17}$$

This expression may be simplified by changing the exponential voltages back to their corresponding opertional form using the relationship

$$\epsilon^{\lambda t}1 = \frac{p}{p - \lambda}1$$

which will give

$$e_{g3} = -e_{g1}\frac{f}{h}\left(\frac{\mu g_p}{C}\right)\left(\frac{-\lambda_1}{\lambda_2 - \lambda_1}\frac{1}{p - \lambda_1} + \frac{\lambda_2}{\lambda_2 - \lambda_1}\frac{1}{p - \lambda_2}\right)\left(\frac{p}{p - \lambda_1} - \frac{p}{p - \lambda_2}\right)1 \tag{18}$$

[6] The operator $\frac{1}{p-\lambda}1$ which appears above when evaluated yields $\frac{1}{\lambda}(1-\epsilon^{\lambda t})1$.

[7] The constants of all stages, though different, give approximately the same response because they are all chosen proportionately.

If this operational expression is multiplied out and similar terms collected, it is possible, by the use of one more operational formula,[8] to evaluate it and obtain

$$e_{g3} = -e_{g1}\frac{f}{h}\,\frac{\mu g_p}{C(\lambda_2-\lambda_1)}\left\{\left[\frac{\lambda_1+\lambda_2}{\lambda_2-\lambda_1}+\lambda_1 t\right]\epsilon^{\lambda_1 t} + \left[-\frac{\lambda_1+\lambda_2}{\lambda_2-\lambda_1}+\lambda_2 t\right]\epsilon^{\lambda_2 t}\right\}1 \tag{19}$$

again substituting $\lambda_2 - \lambda_1 = h$

and $\lambda_1 + \lambda_2 = 2b$

we obtain, finally

$$e_{g3} = e_{g1}\left(\frac{f}{h}\right)^2\left\{\left(-\frac{2b}{h}+\lambda_1 t\right)\epsilon^{\lambda_1 t} + \left(\frac{2b}{h}+\lambda_2 t\right)\epsilon^{\lambda_2 t}\right\}1 \tag{20}$$

which shows the response of the second stage to be the reverse (as regards sign) of the first stage which is the usual case in resistance-coupled amplifiers

In a similar manner the response of the third stage e_{g4} will be found to be

$$e_{g4} = -e_{g1}\left(\frac{f}{h}\right)^3\left\{\left[\frac{2a+4b^2}{h^2}+\frac{\lambda_1 t}{h}(\lambda_2-2b)+\frac{\lambda_1^2 t^2}{2}\right]\epsilon^{\lambda_1 t} + \left[-\frac{2a+4b^2}{h^2}+\frac{\lambda_2 t}{h}(\lambda_1-2b)-\frac{\lambda_2{}^2 t^2}{2}\right]\epsilon^{\lambda_2 t}\right\}1. \tag{21}$$

If the various stages are dissimilar, the above equations, which are the forms for equal roots, will not apply.

A plot of $e_{g1}, -e_{g2}, e_{g3}$, and $-e_{g4}$ as functions of time is shown in Fig. 12. These were calculated from (15), (20), and (21), using the values given in Fig. 9 for the circuit constants. The curves, excepting e_{g1}, are all drawn to have the same maximum height as a convenience for comparison purposes.

For the reception of constant length pulses, such as Arlington signals, the three-stage amplifier is most desirable because the first lobe of the pulse due to starting the wave will be negative, while the first lobe of the pulse due to stopping the wave will be positive. Thus, by properly choosing the constants, the second lobe of the pulse due to

[8] $\frac{p}{(p-1\lambda)^2}1 = t\epsilon^{\lambda t}1$

starting the signal may be made to add to the first lobe due to stopping the signal. This is illustrated in Fig. 13 where the dotted curve shows the resultant voltage effective on the grid of the output tube. If grid detection is employed instead of mutual detection the voltage response in all stages will be reversed.

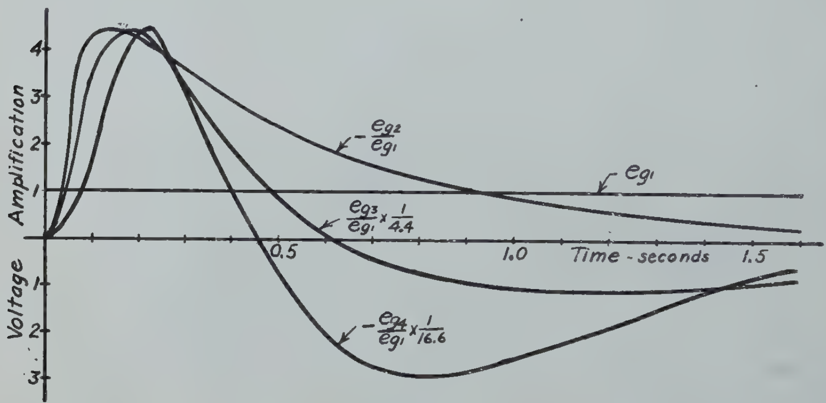


Fig. 12

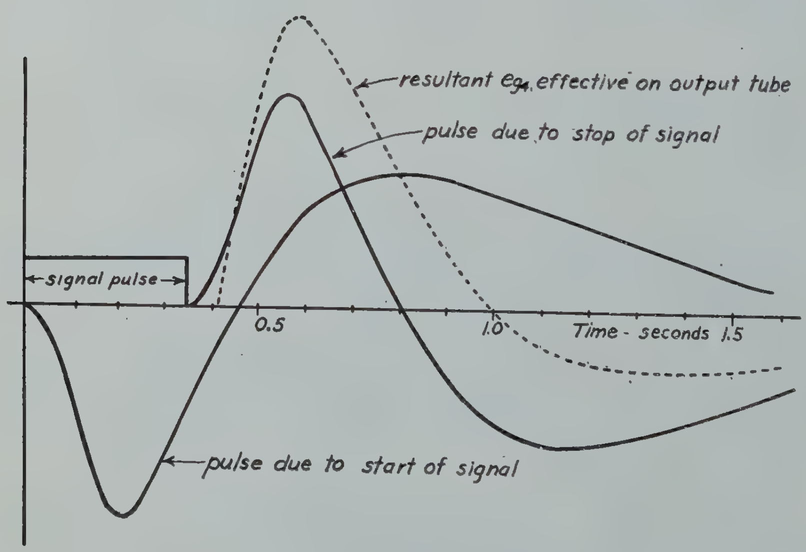


Fig. 13

In actual operation the output tube has a high bias, 7.5 volts, making the normal plate current which flows through the selector magnet quite small, about 0.5 ma. Therefore, the effect of the negative lobe due to starting the signal will be nil while the effect of the positive

lobe will be greatly increased if the signal duration is just right. This last feature gives another safeguard against interference. Extreme positive values, such as at $t = 0.6$ sec. will, when the signals are of normal strength, cause the tube to overload with the result that the peak of the wave will be flattened. This is particularly true in the case of the output tube, which has the dotted voltage impressed on its grid. As the normal negative grid bias of this tube, 7.5 volts, is considerably

Fig. 14

less than the positive signal voltage at $t = 0.8$ sec. the grid will draw current and prevent a large positive grid voltage, producing a flattened plate-current wave. By putting a high resistance in series with the grid of the output tube the plate current of this tube can be further limited over the normal limitation explained above. It is thus possible to make the response of the amplifier nearly the same from some minimum value of signal voltage to any maximum value desired. This feature is desirable as it makes the selector response, within limits, independent of the signal strength.

It is interesting to note that this desirable combination of effects will be produced only with three stages.

The Selector

Even after the time signals are picked up and formed into powerful current pulses by the radio receiver, the problem of correcting a clock is by no means complete. A device that will accurately select the correct pulse to perform the correction is necessary. This device, called a selector, Figs. 14 and 15, is designed to select the first pulse of each minute that follows a five-second period of silence. It works as follows: At about 11:55 A. M. the master clock closes a contact in series with the filaments of the receiver and the 24-volt tap of the power box. In

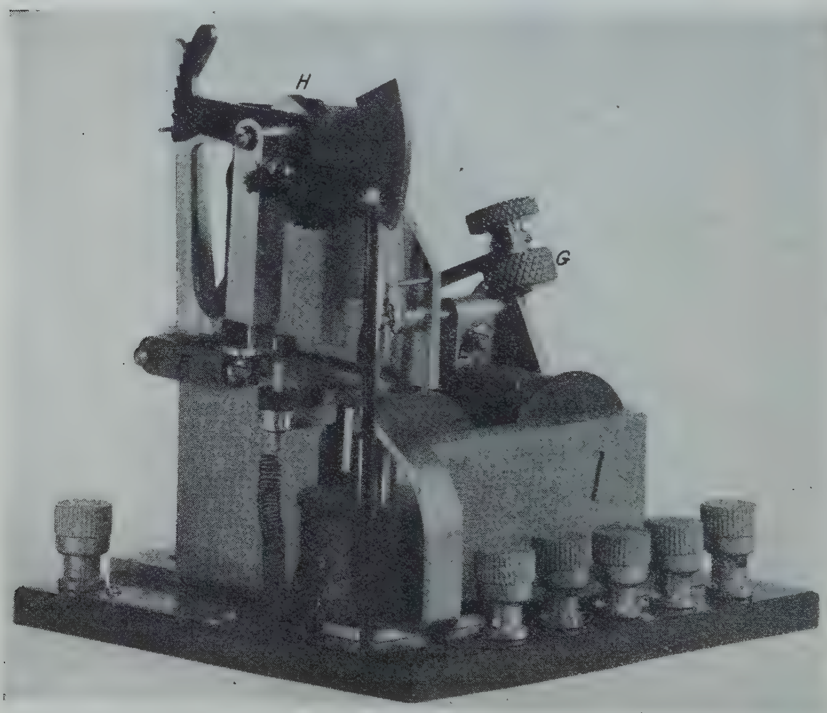


Fig. 15

this same circuit is magnet A, Fig. 14. When this is energized, its armature engages pawls B and C with the teeth of wheel D. The selector is now ready for the time signals.

The bipolar electromagnet is connected in the plate circuit of the output tube, at S, Fig. 2. The first pulse delivered by the receiver causes this magnet to attract its armature E which, in turn, lifts pawl C by means of level F. This advances toothed wheel D up one tooth. It is held in place by pawl B. When the pulse ceases, gravity acting on

a weight G returns the armature and its pawl to their initial positions. This procedure is repeated once each second, until wheel D has been advanced 16 teeth and can go no further due to the lack of more teeth.

The advancement of the last few teeth has caused a cam H on toothed wheel D to withdraw guide I which has been keeping pins J and L from engaging with the teeth on segment K. Pin J is linked to the clock pendulum which causes it to move up and down with a one-second period. In so doing, it pushes segment K down one tooth where it is held by pin L. The next signal pulse causes the magnet armature to force bar M to the left, which disengages pin L from segment K, allowing it to return to its initial position by gravity. This process is repeated until the silent interval is reached. Since the selector magnet is not now operating, pin J forces segment K down five teeth in this silent interval. Projection N on segment K forces contact O to come opposite contact pins P on arm F. The first pulse of the next minute causes contact piece P to touch O, which closes the circuit through the correcting mechanism. At the same time, pin L is knocked out so that segment K returns to its initial position by gravity as does contact O. The correcting mechanism also opens the filament circuit. This releases pawl B so that segment D can also return to its initial position by gravity. The selector thus clears itself and is ready for operation the next day.

It is almost impossible to set the master clock incorrectly. The clock counts off the five seconds in the silent interval and aligns the contact points so that the clock can be set by a pulse only between the 4.5 and 5.5 sec. of the silent interval. If static occurred during this one second interval, the most a clock could be set in error is 0.5 sec. The only other time a false correction could take place would be in the interval between the turning on of the set and the first pulse. To decrease this possibility, the receiver is turned on at as nearly 11:55 A.M. as possible or even a little later. In the case the receiver is turned on a little earlier, the teeth on wheel D make it necessary to receive 15 pulses before a silent interval of five seconds will operate the correcting mechanism. It is practically impossible to receive 15 static pulses that will operate the selector and then have a five-second silent interval followed by another pulse.

The only time that static is harmful is when it occurs in the silent interval. This causes pin L to be thrown out of contact with a tooth on segment K thus spoiling the chances of correcting the master clock during that silent interval. But since there are four chances to correct every day, it very seldom happens that daily correction is not attained.

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A NEW FREQUENCY-STABILIZED OSCILLATOR SYSTEM*

By

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Summary—*A new vacuum-tube self-oscillating system having extraordinary frequency stability is described which depends on the reëntrant circulation of oscillations through tuned filter or coupling units. The reëntrant circulation through the filter sections attenuates all but a single frequency in a manner analogous to the attenuation produced by a filter system having an infinite number of sections. The unattenuated component having a single frequency is amplified at each passage through the system and constitutes the single-frequency oscillation. The methods and necessary precautions for attaining frequency stability are given. Frequency shifts due to ordinary variations of plate potentials, filament current, or keying are found to be of the order of one thousandth of one per cent. The extreme flexibility of the circuits permit the construction of satisfactory radio transmitters operating from the lowest frequencies up to 20,000 kc without the use of frequency-doubling stages. The oscillator system has found wide application in commercial and naval aircraft radio communication problems.*

Introduction

THE ECONOMIC value and importance of constant-frequency radio transmitters under the present congested condition of the ether can hardly be overestimated. The limit to the number of continuous-wave transmitters that can be placed within a given band is determined primarily by the stability and constancy of the transmitted frequency and by the purity of the emitted wave. It is hardly necessary to point out the technical advantages that would result from an oscillator system capable of producing a strictly constant frequency. Such oscillators are not yet available but approximate constancy can be attained. The writer can plainly foresee the time when successful high-frequency channels will be separated by not more than 100 cycles and the art must be developed to a point where such frequency intervals are a commercial fact. Piezo-electric and magnetostrictive constant-frequency oscillators have been the subject of much study in the past few years, and while the idea of "sawing a certain number of cycles out of a quartz crystal or steel ingot" appeals to one's imagination, it leaves a great deal to be desired from many points of view. Oscillators of this type are inflexible and are limited in the frequency range over which they will operate. As a result, modern radio transmitters are greatly complicated by the use of multiple-crystal systems and many

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stages of frequency-multiplying units which add needless complication to the technical problems involved. The inherent disadvantages of piezo-electric and magnetostrictive oscillators led the writer to undertake the development of numerous self-oscillating circuits which are found to be satisfactory constant-frequency oscillators for frequencies up to 20,000 kc.

In 1923 the United States Army Air Service undertook a series of experiments on radio control in which harmonic selectors were employed, and it fell to the writer to provide a suitable audio-frequency modulating source for the control circuits whose frequency would stay satisfactorily constant under the trying conditions that must be encountered in portable aircraft equipment. Ordinary self-oscillating circuits for audio frequencies were entirely out of the question, primarily because of frequency shifts arising from variations in the filament current or emission and from plate-battery fluctuations. Tuning forks were tried but were entirely unreliable and unsatisfactory because of temperature effects and because large accelerations of the aircraft in certain directions tended to stop their vibration.

Various methods were adopted to compensate for frequency shifts in an ordinary oscillator arising from the variation of the plate voltage and filament emission but no truly satisfactory system could be devised. It was recognized early in the work that some new fundamental principle would be necessary if the problem were to be solved in a simple manner. The necessity for such a principle led to a test of an oscillator which was first described by the writer over six years ago, but for reasons which will appear, the system was only of use as a constant-frequency source in the audible-frequency range, and it remained for later development to perfect it for use at radio frequencies.

Causes of Frequency Variation

In any self-oscillating vacuum-tube circuit the accidental changes in the generated frequency arise in general from two major causes. The first, and perhaps the most troublesome, source of difficulty arises from the fact that the generated frequency depends somewhat on the internal tube impedance. If the mean value of this quantity changes for any reason a change in frequency takes place. Changes in the internal impedance of a tube may arise from many sources of which we may name the following as important:

(a) changes in plate potential
(b) changes in the mean grid potential

[1] Gunn, *Jour. Opt. Soc. and Rev. Sci. Inst.*, pp. 545, April, 1924.

(c) changes in filament potential
(d) changes in emission due to causes other than (c)
(e) changes in spacing of the tube elements
(f) keying of the circuit

The second major difficulty is largely mechanical, since it is evident that changes in the mechanical arrangement of any part of the oscillating circuit will change the effective capacities and inductances which in turn will produce a change of frequency of the oscillator. Perhaps the most important causes of these variations are:

(a) changes in temperature
(b) vibration or mechanical forces
(c) electric or magnetic forces

In the common vacuum-tube oscillating circuits employed up to the present time, moderate amounts of energy must be transferred from the plate to the grid circuit for excitation purposes. It is found

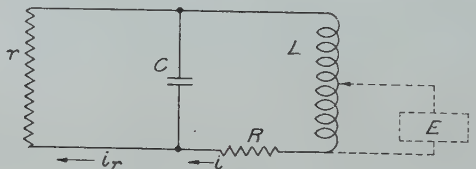

FIG. 1

that the generated frequency produced in such circuits is a complicated function of the internal impedance of the tube and that the law of the variation depends (often critically) on so many factors that compensation by simple means is practically impossible. The circuit theory of the common oscillators taking into account the effect of the tube impedances cannot be worked out here, but in the new circuits, which we shall consider presently, the grid coupling to the succeeding tubes is so small that the excitation circuit can be neglected. The resulting equivalent circuit represented in Fig. 1 is simple, and the frequency relations are readily obtained. In this circuit we shall consider only one symmetrical half of the entire oscillator and take no account of the special properties of the new circuit. In Fig. 1, let r represent the internal plate resistance of the tube which we shall assume has the properties of an ordinary conductor, and let L, C, and R represent the inductance capacity and equivalent series resistance of the associated tuned circuit. The differential equation connecting the circuit constants and the resulting current i for a free oscillation is

$$\frac{d^2i}{dt^2} + \left(\frac{R}{L} + \frac{1}{Cr}\right)\frac{di}{dt} + \frac{1}{LC}\left(1 + \frac{R}{r}\right)i = 0 \qquad (1)$$

This linear equation is solved by the usual methods and it is found that for values of L, R, and C encountered in radio work, the instantaneous oscillating current is given by

$$i = I_0 e^{-\alpha t} \sin \omega t \tag{2}$$

where α is given by

$$\alpha = \frac{R}{2L} + \frac{1}{2rC} \tag{3}$$

and

$$\omega = 2\pi f = \sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{1}{rC}\right)^2} \tag{4}$$

in this expression f is the frequency of the resulting oscillations. Equation (4) may be put in the form

$$\omega = \omega_0 \sqrt{1 - \frac{LC}{4}\left(\frac{R}{L} - \frac{1}{rC}\right)^2}. \tag{5}$$

where ω_0 is 2π times the frequency which would be generated were L and C resistanceless inductances and capacities and were the shunt resistance r absent. Since r represents the internal tube resistance and since, as we have seen, this is a quantity that is subject to change in any practical circuit, the circuit constants should be so chosen that the effect on the frequency of the variations of r are a minimum. Inspection of (5) shows that the effect on the frequency for a change in r is a minimum when r is infinite or if

$$r = \frac{L}{RC}. \tag{6}$$

The internal resistance r of a vacuum tube is not equivalent to the constant resistance we have assumed, since the tube resistance has unilateral conducting properties and is a function of the instantaneous plate potential, but to a crude approximation, (6) specifies the relation that should exist between the various quantities. The mean value of the internal plate resistance of a shield-grid tube can be varied over a moderate range by adjustment of the filament and shield potential which allows a moderately wide selection of the ratio of L to C insofar as this particular requirement is concerned. In any flexible oscillator the relation is not readily satisfied for all adjustments,

but within the limits set by the following design considerations the circuit constants should be so chosen that (6) is approximately fulfilled.

Another factor is still more important in the design of the tuned circuits; for it is clear that the more sharply tuned the elements of a filter chain the more effective is the selective action of the filter. The prime requisite of a simple tuned circuit for such use is that the sharpness of resonance shall be a maximum. That is, the quantity

$$G = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{7}$$

should be made as large as possible. In an earlier experimental investigation,[2,3] relating to the design of high-frequency inductances it turned out that this factor was nearly constant with ranges of L to C varying by as much as two to one, for it was found that the equivalent series resistance of the circuits changed just enough to maintain G approximately constant. Thus considerable latitude is given in the selection of the quantities and for certain adjustments it is relatively easy to satisfy (6) and (7) simultaneously. These adjustments of the electrical circuit to fit the tube aid materially in the production of constant-frequency oscillations but the adjustments alone fail to give the great stability which is readily attained by the method employed in the new circuits and must be considered simply as a refinement necessary only when extraordinary stability is required.

Frequency changes caused by changes in the mechanical positions of the associated conductor systems arise primarily from temperature changes and vibration, although electrical and magnetic forces do occasionally cause trouble. Vibrations of the tube elements can cause difficulty, especially in aircraft, but with better and more rigid tube construction this difficulty will vanish. In a Navy aircraft transmitter employing the circuit to be described, no frequency modulation from vibration was encountered. Perhaps the most troublesome source of frequency shifts arising from mechanical causes originates in thermal expansion and contraction of the parts. The present series of experiments showed that changes in temperature of the tuned circuits might or might not produce changes in frequency, but in no case was the frequency shift of the new oscillator system larger than twice that produced by the same change in temperature in a circuit employing a quartz crystal. Moreover, it was found that the

[2] Gunn, *Radio Broadcast*, page 40, May, 1927.
[3] Gunn, Proc. I. R. E., 15, 797; September, 1927.

geometrical position and arrangement of the circuit elements were important and, by judicious choice, the temperature shifts could be kept to small values. The change in frequency is undoubtedly caused by the expansion of the mechanical parts which changes the effective spacing of the condenser plates, the individual turns of the inductance, etc. It is easy to design special forms of the inductances and condensers which have very small temperature coefficient but this is not necessary for compensation is more readily effected. A simple method has been employed to compensate for changes in frequency due to changes in temperature. The compensating device is simply a small condenser whose capacity is arranged to be a function of the temperature, and consists of a fixed plate and a movable plate made of a stiff bimetallic sheet. This is connected, most simply, across the main tuning condenser, and as the temperature of the device changes the spacing of the plates changes in a manner appropriate to compensate for frequency changes. It is evident that the fractional change of the total capacity can be controlled by the separation of the plates, and that the direction of the change can be reversed by simply turning over the bimetallic plate. Until one gains some experience, the adjustment is rather laborious but it is quite effective and it was found possible in certain cases to keep the frequency constant to within 85 cycles in 15,000,000 when the temperature was changed 10 deg. C. Temperature changes and expansion inside the tube have not been the subject of much study but it seems clear that no particular difficulty will be encountered from this source unless for some special reason series tuning is attempted.

Fundamental Principle of New Circuits

The fundamental idea underlying the new circuits may most easily be understood by reference to Fig. 2, which shows the circuit that the author described some years ago, and is suitable for low frequencies well within the audible range. In this circuit L_1C_1 and L_2C_2 are nearly identical tuned circuits which approximately set the frequency of oscillation. The resultant frequency is also slightly dependent on the resistance of the tuned circuits, on the internal tube plate resistance and on the input coupling units to the succeeding tubes. The circuits L_1C_1 and L_2C_2 are sharply tuned and their parallel impedance is high for the particular frequency to which they are tuned and is low for all other frequencies. Now if a suitable change in potential be applied to the grid of the first tube, the variation is amplified, its phase reversed, and it is passed on to the second grid. The magnitude of the potential passed on the the second grid will depend on the magnitude of the

original change of potential and equally on the apparent impedance of the tuned circuit L_1C_1 since this circuit is effectively across the grid and filament of the second tube. Since the two tubes stand in identical electrical relation to each other, the second tube will repeat the process in precisely the same manner, and the initial pulse will be returned to the first grid amplified or attenuated and approximately in phase with it. When the returning pulse is larger than the initial one it is evident that oscillations will set in, in both the circuits L_1C_1 and L_2C_2 since these will be assumed to be tuned to identical frequencies. The oscillations will be impressed on each grid successively and if the interstage coupling is correct, those frequencies corresponding to a high parallel impedance in the coupling units will be amplified and pass through the system again and again and give rise to a steady oscillation. On the other hand, those frequencies corresponding to a

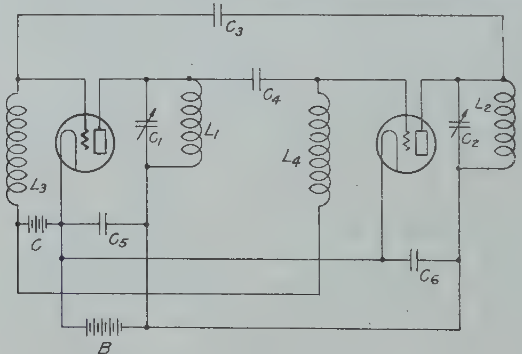

Fig. 2

lower parallel impedance in the coupling units will be less amplified and in the process of passing through the tuned systems again and again will be attenuated to such an extent that they will vanish from the system. It is then easily seen that the reëntrant circulation of oscillations through such a system simulates with great accuracy the selective effect of a filter system having a very great number of sections. Indeed, if oscillations are suppressed in the circuit of Fig. 2 by reducing the plate potential or the interstage coupling, the system makes an extraordinarily selective filter system. It is readily demonstrated that the output from a filter of a great number of sections can be made to approach a single frequency as closely as may be desired by proper design and use of as many sections as required. This demonstration is hardly necessary since the average radio engineer is perfectly familiar with the properties of such filter systems. The present system differs, however, from a filter unit of a great number of sections

in that oscillating energy may be introduced constantly at each section. By proper adjustments and design of the coupling units, which may in themselves be very complicated types of filter sections, the system will oscillate freely at but one frequency and with difficulty, or not at all, at all other frequencies. Actual test shows that this is the case. The first oscillator that was built employing this principle oscillated at 1000 cycles and showed a frequency shift of less than one cycle when the plate potential was changed 50 per cent. Still greater frequency stability can be obtained by increasing the number of sections used, and extraordinary results have been secured by the use of four stages or more.

Application to Radio Frequencies

Initially the circuit failed to show great frequency stability at high frequencies and development on this phase of the problem was temporarily suspended. Later work made it clear that the difficulty with the operation of the circuits at radio frequencies was not with the fundamental idea of reëntrant circulation of the oscillations but was directly traceable to the fact that some of the control oscillation energy was fed back locally through the plate-grid and other stray capacities and couplings and the circulation was not complete. Bridge and balanced circuits were employed to correct these discrepancies and satisfactory results were obtained at moderate frequencies. However, the constant re-balancing and adjustment of the circuits which were necessary when the frequency was changed, robbed the circuit of perhaps its greatest asset——complete flexibility. It has since been found experimentally that any type of circuit that insures the complete circulation of oscillations through the tuned coupling elements again and again will make a satisfactory constant-frequency oscillator at radio frequencies. In the super-frequency band certain circuits and special adjustments become highly desirable. A universal circuit suitable for operation on all but the extremely high frequencies is shown in Fig. 3. In this arrangement screen grid tubes are employed and these have been found most convenient since no balancing adjustment is necessary over nearly the entire frequency range. In this circuit it has been found absolutely essential to shield most carefully the different parts of the system, for it has been found that a slight coupling potential which reaches a grid from its own plate or plate circuit defeats the entire plan of reëntrant circulation of the oscillations. In all the systems to be described the plate circuits are shielded carefully from the control-grid circuits. This is most easily accomplished by incorporating the tuned plate circuit, the coupling

condenser, and the grid choke to the succeeding tube, in one shielded compartment with the tube whose plate belongs to the tuned plate circuit and in another compartment an identical system is set up which works in conjunction with the first (see Figs. 8 and 9). By the careful location of the grid connections between compartments and by careful matching of the two halves, such an arrangement yields an oscillator of extreme stability. On the other hand, if the system is carelessly shielded it is found that energy is invariably led back from the plate circuit of the tube to the grid circuit of the same tube through stray couplings and slight variations produced in the frequency become cumulative. The resulting oscillations are then very sensitive to alterations in the plate and filament power supply.

Referring to Fig. 3 in detail, a pair of electron tubes are shown connected in the oscillator system. Each tube stands in exactly the same electrical relation to its mate and each tube has associated with it

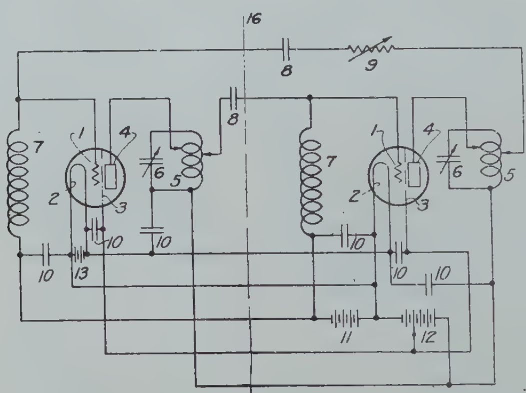

Fig. 3

nearly identical circuits as shown. The circuit elements are represented in the standard manner. The variable capacity, 6, and inductance, 5, constitute a tuned circuit which determines the frequency of oscillation, and must be so chosen that the product of L and C are substantially the same in both circuits. These tuned coupling units are connected to the amplifier-oscillator tubes by means of the coupling condensers, 8, although this is not a necessary arrangement, and coupling may be accomplished by the use of a suitable inductance coupled magnetically to the plate inductance, 5, or by a combination of these methods. When shield-grid tubes are employed the shield is carefully maintained at ground or filament potential by the use of by-pass condensers, 10. This circuit is especially suitable for use as a master oscillator or as a heterodyne frequency meter, but when so used, it should be arranged

to work into a shield-grid output tube whose control grid is coupled to either tuned circuit of the master in the manner indicated in Fig. 8, making sure that the plate and plate circuit of the amplifier are shielded from all the master oscillator circuits. The load is connected to the plate circuit of this output tube and it has been found that changes of any kind in the load produce negligibly small frequency shifts in the master. An essentially similar arrangement is shown in Fig. 4 which makes use of the standard 3-element tubes. With this type of tube it is necessary to balance out the plate-grid back coupling and any standard method can be employed so long as the compensation is carefully carried out. The circuit of Fig. 4 illustrates a typical system whereby the back coupling is compensated by a potential derived from a few overhanging turns and coupling condensers, 15. This circuit has not

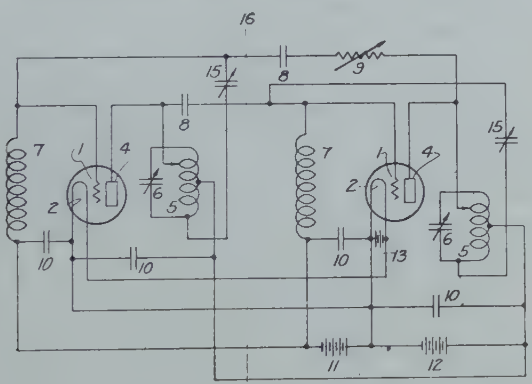

FIG. 4

been found especially convenient for the higher radio frequencies. The variable resistance, 9, is introduced to control the amount of energy fed back to the original amplifier which in turn controls the oscillation current. Perhaps a better manner of controlling the feed back is by a variable tap on the plate coil in the manner indicated in the diagram. It has been found most satisfactory to keep the plate coupling rather large (especially in using shield-grid tubes) and the grid coupling to the succeeding tube as low as possible. The desirability of using low grid coupling arises from the reëntrant circulation of the oscillations which is of such a nature that certain frequencies are amplified and other frequencies are attenuated, the amount depending on the apparent impedance of the tuned circuits and the grid coupling. As the coupling between filter units is reduced the attenuation for all frequencies is increased and a critical point will be found where they all die out. A slight additional coupling brings the level up to a point where only the

one frequency for which the circuits are adjusted is amplified and all others attenuated. This adjustment obviously is the one which gives rise to the most stable oscillations. A circuit which is properly adjusted will not oscillate except at one point on the tuning condenser, and the resonant point is as sharply defined on the condenser scale as in a quartz-crystal oscillating circuit. For example, in a certain circuit operating normally at 10,000 kc, oscillations had ceased completely when one circuit had been detuned to such an extent that the frequency shifted to 10,022 kc or to 9,977 kc.

Performance

In order to determine what factors produced the greatest changes in frequency a series of standardization tests were run on an oscillator of the type shown in Fig. 3, and operating at 15,018 kc. These tests showed that the circuit is extraordinarily stable and the frequency stability of the oscillator compares favorably with a piezo-electric crystal-controlled oscillator maintained at a constant temperature by means of a thermostat. Rather complete tests have been conducted on the effect of changes of plate and filament potential on the frequency, and on the lilt produced by keying in different parts of the circuit. Changes in plate potential of 10 per cent produced a change of frequency of but 45 cycles. This change in frequency amounts to but 0.0003 per cent of the fundamental and is so small as to cause no difficulty whatever in any normal radio circuit. A curve showing how the frequency changes with the plate potential is given in Fig. 5. Perhaps the greatest changes in frequency are produced in a vacuum-tube oscillator by changes in the filament potential. With the present circuit such changes are relatively small and amounted to but 400 cycles for a change in filament potential of 8 per cent. A curve typical of the performance in this respect is given in Fig. 5(*b*). Keying of the entire circuit in the plate lead gave rise to no lilt that could be detected until the frequency was pushed up to 20,000 kc, and even then the shift in frequency due to keying was just noticeable. The foregoing data are typical of the performance that may be expected from an oscillating system similar to Fig. 3, when properly adjusted to the desired frequency. It has been found that the operation of such an oscillator is subject to a very small drift in frequency over long periods of time due to permanent changes in the filament emission unless the precautionary adjustment specified by (6) is made. A curve showing how the frequency changes with the circumambient temperature is given in Fig. 5(*c*). In this particular run no attempt was made to compensate for temperature changes, but over a narrow range of a degree or so the

frequency changed but slightly and this region would ordinarily have been selected for operation. The shape of this curve depends markedly on the particular arrangement of the parts, and Fig. 5(c) is niven only as an illustration of what may be expected. When the gircuits have been in adjustment the writer has never observed changes ic frequency with temperatures which amounted to more than 60 parts per million per degree. By compensation this can very easily be reduced to about 10 parts per million per degree over a range of two or three degrees. In special cases the change was as small as one-twentieth of this value. The present circuits are therefore comparable in stability to a piezo-electric oscillator in respect to temperature variations, for

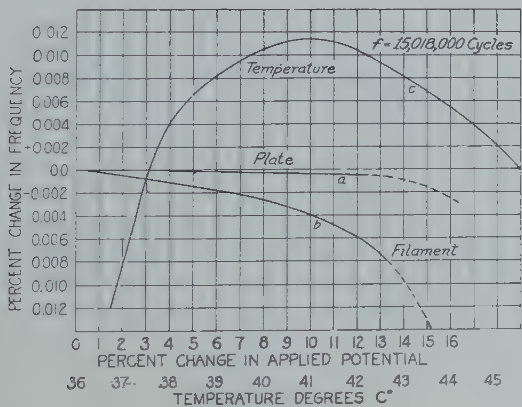

FIG. 5

crystal oscillators show changes of about 22 parts per million per degree.

It should be emphasized at this point that the above performance data apply to a laboratory oscillator whose power supply for all parts of the system was steady direct current. Moreover, as has been found in other types of constant-frequency oscillators, it is essential that the tubes be lightly loaded when great stability is required. Dr. A. Hoyt Taylor of this laboratory has subjected the circuit to many tests in his transmitting laboratory and has not found the circuit quite as stable as the foregoing paragraph indicates. In his tests the tubes were worked at their rated output, and alternating current was supplied to the filaments. The effect of alternating current on the filaments was found especially unfavorable since the internal impedance of the tube varied over a considerable range at a frequency twice that of the heating current. This variation can undoubtedly be cured by the use of special non-inductive filaments but so far special tubes have not been made up.

In general, it is recommended that the circuit be worked lightly and any power that is required be taken from a power amplifier associated with the master. When the master oscillator is keyed in preference to the power amplifier, it is still more desirable to work the system lightly, for thermal equilibrium is never attained under such operating conditions.

Special Circuits

Many variations of the fundamental circuits are possible without departing from the basic principle of reëntrant circulation which property gives rise to great frequency stability. The circuit of Fig. 6 is a

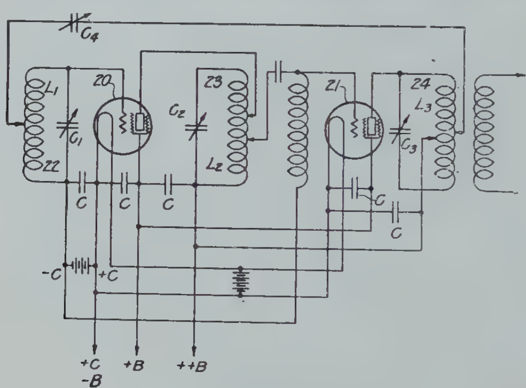

Fig. 6

convenient arrangement when a power amplifier, 21, is to be used in connection with a small master-oscillator tube, 20. The circuit has not been found quite so stable as the circuit of Fig. 3, since large changes in the load on the power amplifier changes appreciably the effective circuit constants of the tuned circuit L_3C_3 and the generated frequency is slightly changed. The change in frequency is kept small by designing the tuned circuits L_1C_1 and L_2C_2 so that these circuits are very sharply tuned. These two circuits are depended on primarily to stabilize the circuit by their filtering action, and the circuit L_3C_3 serves only as a broadly-tuned coupling unit to the load and to the input circuit of the master tube.

A degree of stability may even be attained by the use of a single tube. Such an oscillator shown in Fig. 7 is essentially a tuned-plate tuned-grid oscillator circuit with a link coupling circuit $L_{1c}L_{2c}C_2$ which also serves as a tuned filter section. The three tuned circuits are all adjusted to substantially the same frequency and oscillations circulate continuously through the system. Complete circulation is insured by

carefully reducing the plate—control-grid capacity coupling to an absolute minimum. This is accomplished by the use of a screen-grid tube and by placing a comb-like electrostatic shield made of copper wires between the plate coupling coil and the inductance L_{2c} with a

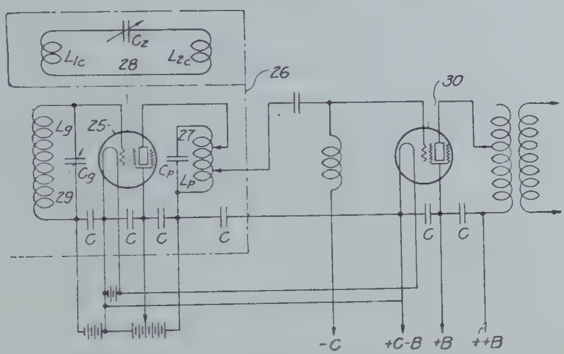

FIG. 7

similar shield between the grid coupling coil and the inductance L_{1c}. Any residual electrostatic coupling can be partially neutralized by slight grid coupling back to a few overhanging turns of the plate coil L_p. The circuit is not particularly recommended for ordinary use as it is

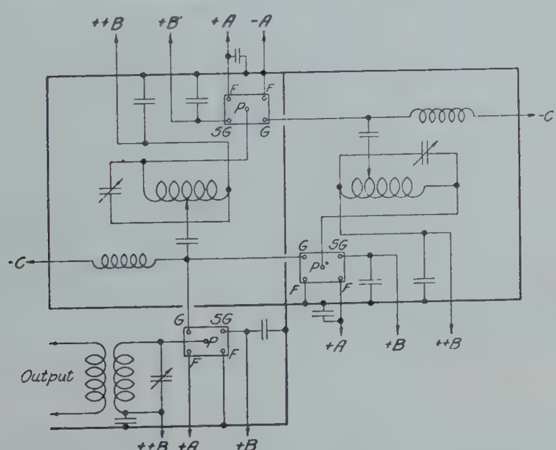

FIG. 8

very difficult to secure the stability which is readily attained with the other multiple tube circuits here described.

An examination of the operation of the circuit given in Fig. 3 shows that the potentials of the two control grids are about 180 deg. out of phase when the two halves of the oscillator are nearly identical and the

oscillating currents in each are the same. The oscillator then makes an ideal master for a push-pull amplifier as the control grids of the two amplifier tubes are simply coupled to the two plate circuits by any suitable means and the correct phase relations are immediately established. A word of caution may not be amiss in this connection. It is to be noted that unless the circuits are laid out carefully the coupling to the push-pull amplifiers may serve to couple the two oscillator units and much of the frequency control may be lost due to the lack of complete circulation. It has been found desirable to shield the grid circuits of the two push-pull amplifiers from each other and by careful adjustment make sure that the radio-frequency ground potential of the two amplifier filaments is exactly the same as that of the oscillator filaments. At the highest frequencies this may be difficult.

Conclusion

The circuits which have been described all make use of the reëntrant circulation of oscillations in such a manner that oscillations of a single frequency are produced. A study of actual oscillator performance has shown that the new circuits when properly adjusted are capable of producing oscillations which are as stable as other oscillator systems employing tuned mechanical systems. The new system has the advantage of extreme flexibility and has been found to be valuable in naval aircraft radio communication problems. It should find broad application wherever a flexible and stable oscillator is required, but its flexibility probably precludes its use as a primary standard of frequency.

Proceedings of the Institute of Radio Engineers
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INTERPOLATION METHODS FOR USE WITH HARMONIC FREQUENCY STANDARDS*

By

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Summary—*Interpolation methods for determining the value of an unknown frequency in terms of harmonic standard frequencies are discussed under the following classifications:*

I. Direct beating methods, wherein the beat between known and unknown frequencies is utilized directly to operate frequency indicators or measuring devices.

II. Direct interpolation methods in which the fundamental frequency of an interpolation oscillator is adjusted to zero beat in turn with the unknown frequency and the adjacent known harmonic frequencies. The unknown frequency is then found from the oscillator dial readings.

III. Harmonic interpolation methods which are an extension of the principles of (II) permitting an interpolation oscillator of limited fundamental frequency range to be employed in the measurement of frequencies lying both above and below this range.

IV. The principles of (III) point to a means for covering a wide range of unknown frequencies through the use of a low-frequency narrow-range oscillator fitted with harmonic producing circuits. A greatly opened-out interpolation scale may be obtained.

Some disadvantages and limitations of the various methods are considered, as well as some advantages.

Introduction

METHODS for deriving from a single-frequency standard oscillator (such as a piezo-electric oscillator) a large number of frequencies, related to the frequency of the standard by the ratio of two integers, have previously been described[1] and are now widely used. Circuits may be arranged for either frequency multiplication or division, or a combination of both, so that the harmonic frequencies derived from a single-frequency standard of frequency f_s may be written as

* Dewey decimal classification: R213. Original manuscript received by the Institute, May 15, 1930. Presented before the Washington Meeting of the U.R.S.I., April 25, 1930.

[1] W. A. Marrison, "A high precision standard of frequency," Proc. I.R.E., **17**, 1103; July, 1929. L. M. Hull and J. K. Clapp, "A convenient method for referring secondary frequency standards to a standard time interval," Proc. I.R.E., **17**, 252; February, 1929. D. W. Dye, "A self-contained harmonic wavemeter," *Phil. Trans. Roy. Soc.* 224, A, 279. Balth. van der Pol, "On relaxation oscillations," *Phil. Mag.* 978, November, 1926. J. K. Clapp, "Universal frequency standardization from a single frequency source," *Jour. Opt. Soc. Am. and Rev. Sci. Instr.* **15**, 25; July, 1927 (with bibliography).

$$f = \left[\frac{f_s}{m}\right] n \tag{1}$$

where n and m are two integers, n being the order of multiplication and m the other of division.

By such methods frequencies known in terms of the standard frequency may be obtained in the region of practically any unknown frequency. The problem confronting us in this paper is that of determining the difference in frequency between the unknown frequency and a convenient known frequency near it. Since the harmonic standard sources provide a series of known frequencies at equal intervals on the frequency scale, the problem may also be considered as one in interpolation between two known frequencies.

These viewpoints are indicated in the diagram of Fig. 1, in which the unknown frequency f_x is indicated as lying between the nth and

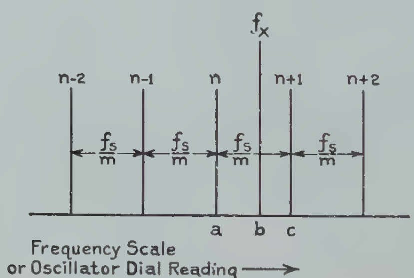

Fig. 1—The diagram indicates the relationship of an unknown frequency f_x to the harmonics of a standard frequency of base frequency f_s/m. The frequency difference ab or bc is to be determined, methods for so doing being given in the text.

$(n+1)$th harmonics of a standard harmonic source of fundamental frequency $[f_s/m]$ where f_s is the frequency of the controlling oscillator, and m is the order of frequency division. If the differences

$$f_x - n\left[\frac{f_s}{m}\right] \quad \text{or} \quad (n+1)\left[\frac{f_s}{m}\right] - f_x \tag{2}$$

(ab and bc respectively) are determined, then the unknown frequency is determined. Since the interval between any adjoining pair of harmonics is equal to the base frequency $[f_s/m]$, it is many times more convenient to express the unknown frequency in terms of the ratio of the frequency difference between it and one of the adjacent harmonics to the interval between harmonics,

$$\text{i.e.,}\quad f_x = n\left[\frac{f_s}{m}\right] + \frac{ab}{ac}\left[\frac{f_s}{m}\right] = (n+1)\left[\frac{f_s}{m}\right] - \frac{bc}{ac}\left[\frac{f_s}{m}\right]$$

or, in terms of the linear interpolating oscillator condenser scale readings

$$f_x = n\left[\frac{f_s}{m}\right] + \frac{\theta_x - \theta_n}{\theta_{n+1} - \theta_n}\left[\frac{f_s}{m}\right] = (n+1)\left[\frac{f_s}{m}\right] - \frac{\theta_{n+1} - \theta_x}{\theta_{n+1} - \theta_n}\left[\frac{f_s}{m}\right] \tag{3}$$

(Shown graphically in Fig. 4).

Many methods for carrying out such measurements have been suggested, among them those summarized below. These methods group into a few fundamental classifications according to the basic principles employed in determining the frequency differences, such as direct beating, direct interpolation, and harmonic interpolation. Some of the more convenient methods of each class will now be considered.

I A. Direct Beating Methods

In these methods voltages of the unknown frequency and of the known frequency are impressed on a detector and the value of the resulting difference beat frequency is determined by methods such as the following:

(a) Matching beat frequency with the frequency of an adjustable calibrated oscillator by means of telephones, oscillograph, or beat indicator. The value of the beat frequency is then determined from the oscillator calibration. (Fig. 2).

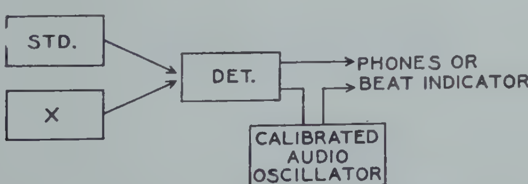

Fig. 2.—A known frequency from the standard source (STD) and the unknown frequency (x) are impressed on a detector. The difference beat frequency resulting is matched by a frequency from a calibrated audio oscillator, the matching being indicated by telephones as beat indicator.

(b) The difference beat frequency may be impressed on a frequency bridge,[2] or on any one of various resonant circuit arrangements

[2] G. W. Pierce, *Proc. Am. Acad. of Arts & Sci.* **63**, 1; April, 1928.

associated with resonance indicators, such as vacuum tube voltmeters. (Fig. 3). The value of the beat frequency is then determined from the bridge or resonant circuit calibrations.

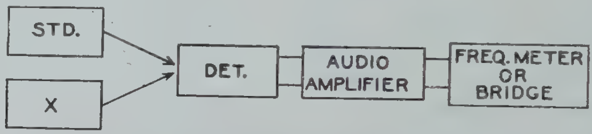

Fig. 3—The known and unknown frequencies are impressed on a detector and the difference beat frequency resulting is amplified sufficiently to operate a direct-reading frequency meter, a frequency bridge, tape recorder, or similar devices.

(c) The difference frequency may be impressed on an impulse counter, or a high speed recorder, for a known time interval and its value determined from the indicated reading of the counter or from the tape record of the recorder.

(d) Use may also be made of direct reading frequency meters of either the pointer or multireed type.[3]

(e) The audio-beat frequency may be compared *with standard audio frequencies derived from the same standard control frequency as the radio-frequency harmonics.* This extension of the methods (a) and (c) above very greatly increases the possible precision of measurement, as the audio-frequency oscillator (as employed in (a)) may be used as a comparator, i.e., it may be calibrated as it is used, so that no dependence need be placed on its calibration over long intervals of time.

By comparing the beat frequency with an appropriate standard audio frequency, the difference between the two may be made much smaller than either of the audio frequencies. This makes it possible to detect easily small changes in the frequency under measurement. By suitable choice of the standard audio frequency employed, it is possible always to keep the difference between it and the beat frequency below a certain desired value. This is particularly useful when a recording mechanism having a definite maximum response frequency is used.

I B. Limitations of Direct Beating Methods

The methods given in the preceding section are open to some serious limitations in general application; these limitations do not always apply in special applications.

When the beat frequency lies in the normal audio-frequency range,

[3] R. C. Hitchcock, *Proc. I.R.E.*, 17, 24; January, 1929.

from about 200 to 5000 cycles per sec., aural methods may be employed. These methods in general require least amplification and are consequently more convenient in application.

For values of the beat frequency above the useful aural range, resort must be made to beat indicators such as vacuum tube voltmeters.

If the difference frequency lies below about 200 cycles per sec., resort must be made to the "visual or mechanical" methods—oscillographs, counters, recorders, or direct reading frequency meters. These usually require a substantial amplification of the beat frequency, particularly if measurements of the frequencies of distant transmitters are desired.

The general accuracy of measurements by these methods depends upon such factors as the accuracy of calibration of the oscillators and frequency bridges or upon the accuracy with which a record may be read. For all but the most precise measurements the precision of adjustment is sufficiently high.

When the difference beat frequency is compared with a standard audio frequency derived from the same standard frequency assembly as that employed for producing the standard harmonic radio frequencies, the precision of measurement of the beat frequency may be very greatly increased. The method necessitates more elaborate equipment and considerable care in interpretation of results.

II A. Direct Interpolation Method

In this method, a radio-frequency oscillator, adjustable through the region of the unknown frequency, is set to zero beat* with the unknown frequency and the adjacent standard harmonic frequencies in turn. For convenience in reducing computations, the oscillator should have a linear frequency calibration. The value of the difference in frequency between the unknown and either standard frequency is

* The author again wishes to call attention to a convenient and rapid method for adjusting two radio-frequency oscillators to zero beat by an aural method. The method was described some time ago (*Exp. Wireless & Wireless Eng.*, Editor's letter, p. 174, March, 1927) but has not been much used in laboratories in this country. If, in any of the assemblies here mentioned, the detector is made to oscillate at a frequency slightly different from that at which the two oscillators are to be adjusted in zero beat, a tone will be heard in the detector output. This tone will wax and wane in *intensity* in accordance with the difference in frequency of the two oscillators which should be in zero beat. Changing the pitch of the detector output tone, by variation of the detector frequency, does not alter the period of the waxing and waning. Adjustment of one of the oscillators will then increase the period of waxing and waning, until one cycle of this variation in intensity may be several seconds long, even when the oscillators are operating on frequencies of a few million cycles per second, representing a difference in frequency of a fraction of a cycle per second.

then determined from the scale readings of the oscillator, without regard to its absolute frequency calibration. The functional diagram is given in Fig. 4. The method has many advantages for rapid measurements and also does not require an unusual amount of amplification.

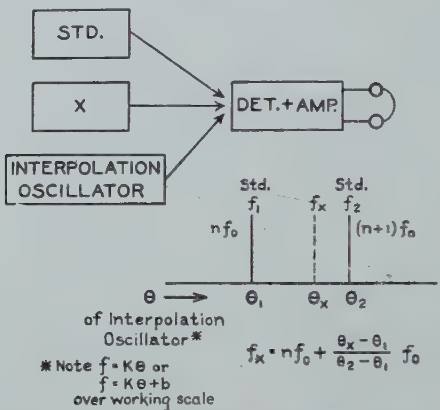

Fig. 4—Illustrating the use of an adjustable radio-frequency oscillator for interpolation between two known frequencies, such as adjacent harmonics of an harmonic standard frequency source. The interpolating oscillator is adjusted to zero beat in turn with each of the known frequencies and with the unknown frequency. The unknown frequency may then be determined from the known standard frequencies and the interpolating oscillator dial readings.

II B. Limitations of Direct Interpolation Method

The accuracy of the method depends upon the accuracy with which the linear calibration of the variable-frequency oscillator may be realized and maintained and upon the slope of the calibration curve. If the slope is too great, the successive readings of the oscillator dial fall very close together, so that good accuracy is not possible in taking the differences of these readings. The accuracy also depends upon the value of the base frequency of the standard harmonic series. If this be low, only a small change in the interpolation oscillator frequency is required to pass from one harmonic of the standard to the next. The errors caused by the curvature of the variable frequency oscillator characteristic are then much smaller than those which exist when a larger frequency change is required, i.e., when the base frequency is high.

A serious disadvantage exists in the practical application of the method if a wide range of frequency is to be covered by the interpolation oscillator, since many coils may be required.[4] As a case in point, a suitable oscillator for making measurements in the broadcast frequency band (500 to 1500 kc) requires the equivalent of at least six coils, in order that interpolations may be made to the nearest five cycles on a standard harmonic series having a base frequency of 10 kc.

III A. Harmonic Interpolation Method

Instead of attempting to adjust the fundamental frequency of the interpolation oscillator over a wide range, use may be made of harmonics of the frequency under measurement, or harmonics of the interpolation oscillator, to extend greatly the useful range of a given interpolation oscillator. In this manner it is necessary to design the interpolating oscillator for adequate performance over only a limited fundamental frequency range.

For extension to lower frequencies, a harmonic of the unknown frequency is measured by the method outlined in II A. The unknown frequency is then

$$f_x = \frac{n\left[\frac{f_s}{m}\right] + \frac{\theta_x - \theta_n}{\theta_{n+1} - \theta_n}\left[\frac{f_s}{m}\right]}{n_x} = \frac{(n+1)\left[\frac{f_s}{m}\right] - \frac{\theta_{n+1} - \theta_x}{\theta_{n+1} - \theta_n}\left[\frac{f_s}{m}\right]}{n_x} \tag{4}$$

where θ_x, θ_n, and θ_{n+1} are respectively the linear interpolating oscillator dial readings when zero beat is obtained with the harmonic of the unknown frequency and the adjacent harmonic standard frequencies, and n_x is the harmonic number of the frequency being measured. n_x is often known, but if it is necessary to determine it, this is readily accomplished through the use of the interpolating oscillator, thus: Having made the determination indicated above, adjust the interpolating oscillator to zero beat with the next higher or lower harmonic of the unknown frequency, that is, to

$$(n_x + 1)f_x \text{ or } (n_x - 1)f_x. \tag{5}$$

If a calibration of the interpolator is available, the difference in frequency corresponding to the difference between the first and either of the second determinations is f_x and this determination is usually sufficiently accurate to determine n_x, since n_x must be a whole number.

If no calibration is at hand for the interpolator, the frequencies corresponding to the next lower or higher harmonic of the unknown frequency must be determined in terms of the standard harmonic series, just as the frequency of the first harmonic used was determined. (Equation (3)). The difference between either of these frequencies and that of the harmonic first used is, of course, the unknown fundamental frequency, but as this value is obtained by subtraction the accuracy is not as high as may be obtained by using this approximate value to find the number of the harmonic first used and thence the unknown fundamental frequency, thus:

$$(n_x + 1)f_x - n_x f_x = f_x$$
or
$$n_x f_x - (n_x - 1)f_x = f_x \tag{6}$$

(where the left-hand terms are found by (3)).
Then

$$\frac{(n_x f_x)}{f_x} = n_x \tag{7}$$

(where $(n_x f_x)$ was found from (3) and f_x from (6)). n_x must be integral. The experimental values placed in (7) may not yield an integral value for n_x, but they will indicate without doubt the appropriate integral value which would be obtained if there were no errors. Both quantities of the left-hand side of (7) are *experimentally determined;* by choosing the appropriate *integral value* for the harmonic number n_x and rearranging,

$$\frac{(n_x f_x)}{n_x} = f_x \tag{8}$$

we have only one experimentally determined quantity, ($n_x f_x$ from (3)) permitting the value of the unknown frequency f_x to be calculated to the full number of significant figures available in the measurement of the harmonic frequency, $n_x f_x$.

For extension to frequencies above the range of the interpolating oscillator, use may be made of the harmonics of the interpolator. One of these is brought to zero beat with the unknown frequency, and the dial reading θ_x corresponding is noted. The dial readings corresponding to the standard harmonic frequencies next above and below are then determined. The unknown frequency is then

$$f_x = n_x\left[n\left(\frac{f_s}{m}\right) + \frac{\theta_x - \theta_n}{\theta_{n+1} - \theta_n}\left(\frac{f_s}{m}\right)\right]$$
$$= n_x\left[(n + 1)\left(\frac{f_s}{m}\right) - \frac{\theta_{n+1} - \theta_x}{\theta_{n+1} - \theta_n}\left(\frac{f_s}{m}\right)\right] \tag{9}$$

where n_x is the order number of the interpolator harmonic employed. If other considerations do not fix the value of n_x, it may be determined by use of the interpolator as follows:

After having made the adjustments noted above, readjust the interpolator so that the next lower or next higher harmonic is brought to zero beat with the unknown high frequency. The value of the fundamental frequency corresponding is then determined. If the interpolator fundamental frequency is f in the first case and f_1 or f_2 in the second case, then

$$\frac{f}{f_1} = \frac{n_x}{n_x - 1} \text{ or } \frac{f}{f_2} = \frac{n_x}{n_x + 1} \tag{10}$$

So

$$n_x = \frac{1}{1 - \frac{f_1}{f}} = \frac{1}{\frac{f_2}{f} - 1} \tag{11}$$

determining n_x, since f and f_1, or f and f_2 are known.

Again, the experimental values for the frequencies inserted in (10) or (11) may not yield an integral value for n_x. Unless n_x is very large, however, the experimental results will indicate the appropriate integral value of n_x. This integral value should of course be used in (9).

III B. Limitations of Harmonic Interpolation

The same considerations apply as given under II B for the accuracy of the method. The extension to lower and higher frequencies involves an effective division or multiplication of the measured frequency. This extension does not alter the *percentage* accuracy.

IV A. Use of Narrow Range Interpolation Oscillator

A special form of interpolator may be made up on the basis of the extension to higher frequencies noted in Section III A. (See Fig. 5). In this arrangement, a narrow range low-radio-frequency oscillator is used with harmonic generating equipment for emphasizing the oscillator harmonics up to the order of the 150th. Consideration of the range of fundamental frequency variation in the oscillator, the orders of harmonics employed, and the range of frequency over which measurements are to be made will show that for a continuous sweep of the oscillator harmonics over the desired working range (no gaps), $n\Delta$ must be equal to or greater than unity, where n is the order number of the *lowest* oscillator harmonic to be used and Δ is the fractional frequency range of the oscillator, i.e.,

$$\Delta = \frac{f_{\text{max}} - f_{\text{min}}}{f_{\text{min}}}.$$

As an example, a 10-kc oscillator having a range of 250 cycles variation may be used to sweep the broadcast band (500 to 1500 kc) with the harmonics lying between the 50th and 150th. To cause the 50th harmonic to change by 10 kc, the fundamental frequency must be varied the full 250 cycles, which may be made the entire range of the working scale of the oscillator control. The 150th harmonic is then

changed by 10 kc by altering the oscillator control by one-third of its scale.

The method of interpolation between harmonic standard frequencies is identical with that given under II A, as long as the *same*

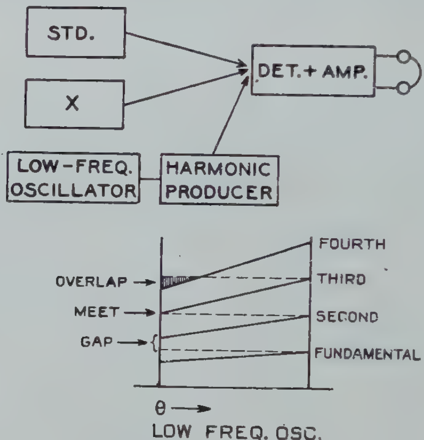

Fig. 5—Illustrating the use of a low-frequency narrow range oscillator with harmonic producing equipment as an interpolating system giving a continuous sweep over all portions of the useful frequency range. In the diagram the frequency variations for the fundamental, second, third, and fourth harmonics are shown. For all harmonics above the third, the range of each overlaps the preceding, giving a continuous coverage for interpolation purposes for all frequencies above the lowest value of the second harmonic.

oscillator harmonic is brought to zero beat with each of the known and unknown frequencies in turn. Under this condition it is *not* necessary to know *which* harmonic of the oscillator is being used.

IV B. Limitations of Narrow-Range Method

Carried to an extreme, the expansion of a 10-kc variation in output frequency to occupy the entire sweep of the oscillator control, imposes rigorous requirements on both the linearity of calibration of the oscillator and upon its stability with time. Since these requirements are difficult to meet, the improvement in accuracy which should result from the more open interpolation scale is not fully realized.

Conclusions

In conclusion it may be suggested that certain applications of the methods outlined above are particularly adapted to the type of measurements listed below:

(a) General laboratory and experimental measurements

These are generally most easily carried out by use of direct

beating methods utilizing a calibrated audio oscillator or a frequency bridge for obtaining the value of beat frequency, within the range where such measurements are possible. Outside of this range the direct interpolation and harmonic interpolation methods give very sátisfactory results over a wide range of frequencies and with simple and easily manipulated equipment.

(b) Measurement of specific frequencies

These measurements involve the routine checking of a limited number of frequencies. In such cases the harmonic-interpolation method is usually simplest and yields results which are sufficiently accurate for commercial requirements.

(c) Precision measurement

The methods most suitable depend largely upon the immediate problem and no general conclusions can be laid down. The extension of the direct-beating method offers a means of precise measurement of frequency with relatively simple equipment and also offers a means for detailed study of small frequency changes when some form of recording equipment is employed for indicating the changes in the output audio frequency.

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A PRECISE AND RAPID METHOD OF MEASURING FREQUENCIES FROM FIVE TO TWO HUNDRED CYCLES PER SECOND*

By

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Summary—*After a brief discussion of some of the common methods of measuring frequencies from 0 to 200 cycles, a method is described which offers important advantages, combining a high degree of accuracy with ease and rapidity of use. The method depends on the fact that if a condenser be discharged through a resistance f times per second (the condenser being charged to the same initial voltage each time), then the average voltage drop across the discharging resistance is directly proportional to f. The unknown frequency is made to control the number of times per second the condenser is discharged, and thus the voltage drop mentioned above is proportional to the unknown frequency. A circuit arrangement is described whereby this voltage drop is balanced, through a sensitive galvanometer and high resistance, against a known fraction of the total voltage drop along a slide-wire resistance shunted around a storage cell. By first calibrating the system with an alternating current of known frequency, it is possible to read unknown frequencies directly off the slide wire.*

A discussion of the sources of error, and experimental determinations of the error actually observed, lead to the conclusion that, in the range from 5 to 200 cycles per second, the accuracy is always better than one-tenth cycle.

I. Introduction

During the course of the work carried on by the Bureau of Standards in the development of secondary standards of frequency, it is necessary to make daily measurements of the beat frequencies between various piezo oscillators used as secondary standards. As some of these secondary standard piezo oscillators do not vary in frequency from day to day by more than a few tenths of a cycle per second, the method of measuring the beat frequencies should be accurate to better than a tenth of a cycle per second. The frequency range of the beat notes to be measured is usually from 0 to about 100 cycles per sec.

One method of measurement used is to record the beat frequency photographically on an oscillograph, at the same time impressing second signals on the record from a standard clock. As the constancy of the piezo oscillators improved, however, it became apparent that this method required too much time to be practicable. In order to obtain an accurate record by this means, it is necessary to let each

* Dewey decimal classification: R210. Original manuscript received by the Institute, March 25, 1930.

exposure run for about 20 seconds. This gives a record which is very arduous to count, and the number of measurements to be made is such that the complete process of intercomparing all the piezo oscillators and reducing the results occupies too much time.

II. Early Work

Consideration was given to the possibility of using a direct-reading frequency bridge, special consideration being given to the modified form of Hay's bridge, as described by Soucy and Bayly.[1] This arrangement was not found to be entirely suitable for the purpose, however, for two reasons: first, in order to secure adequate sensitivity in the region of very low frequencies used, it would be necessary to use an unreasonably large air-core inductance, and second, as an aural balance would not be sufficiently accurate, because of the low sensitivity of the ear at these frequencies, it would be necessary to use a tuned vibration galvanometer, which is not considered practicable in view of the wide range over which it would have to be tuned.

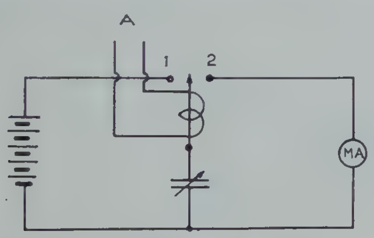

Fig. 1—Circuit arrangement for measuring frequency by means of discharge of a condenser through a milliammeter.

The arrangement presented in this paper is an outgrowth of a system originally described by Maxwell[2] for rough measurements of capacity. Fig. 1 is a circuit diagram of the arrangement as altered by Fleming and Clinton.[3] The alternating current, the frequency of which is to be determined, is fed into a polarized relay at *A*. On one-half of each cycle the contactor is drawn to contact 1, charging the condenser to the voltage of the battery. On the other half of each cycle, the contactor moves to contact 2, discharging the condenser through the milliammeter. Obviously, as long as the time constants of the circuits are kept low enough so that charge and discharge are complete

[1] Soucy and Bayly, "A direct reading frequency bridge for the audio range, based on Hay's bridge circuit," Proc. I.R.E., **17**, 834; May, 1929.
[2] Maxwell, "Electricity and Magnetism," Section 775, Volume II.
[3] Fleming and Clinton, "On the measurement of small capacities and inductances," *Proc. Phys. Soc.* (London), **18**, 386. See also F. A. Laws, "Electrical Measurements," p. 373, 1917.

within the limits of error of the measurement, the quantity of electricity which passes through the milliammeter in a given time interval depends solely on the number of times the condenser is discharged in that interval, and, hence, on the frequency of the alternating current applied to the polarized relay. Therefore, the reading of the milliammeter is directly proportional to the frequency. In practice, of course, the instrument has to be calibrated by applying a known frequency, say, 100 cycles per sec., to the polarized relay, and then adjusting the battery voltage and the capacity until the meter reads 100. Then if some other frequency is applied to the relay, the milliammeter reading will give the frequency directly. The accuracy, however, is only of the order of one per cent of the full-scale reading of the milliammeter, which, while sufficient for many laboratory purposes, was not great enough for the measurements required.

III. Final Design

It was recognized that the sensitivity of the above arrangement could be made amply sufficient by converting it to a null method and

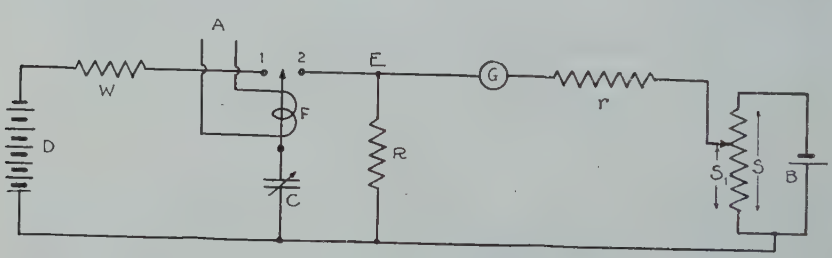

Fig. 2—Schematic diagram of circuit arrangement used for measuring low frequencies.

using a sensitive galvanometer. It was desirable also to have the readings directly in frequency. The circuit arrangement finally used is shown in Fig. 2. The voltage divider S is a slide-wire, with a thousand divisions on the scale, and accurate to the nearest quarter division at any point on the scale. The voltage is measured across the resistance R, through which the condenser C discharges. The resistance r is inserted to serve a double purpose; to keep the galvanometer from being greatly overdamped, since R and S are both of low resistance; and, more important, to limit the current that is drawn from the slide-wire. At first sight this appears to be an anomalous condition, since the adjustment is made to give zero reading on the galvanometer, but it must be remembered that a continuous voltage is being balanced against the average value of a pulsating voltage. This pul-

sating voltage is relatively very high during a very short part of the pulsation period, and is practically zero for most of the period. Therefore, there is only one instant during each pulsation when the galvanometer current is actually zero. Hence, a sufficiently high resistance must be used in series with the galvanometer to limit the current to a very small value, in order that the settings of S_1 may indicate the true relative potentials.

In order that a small variation in S_1 may given an appreciable deflection, a high sensitivity galvanometer should be used. As the accuracy of calibration of the slide-wire is about two and a half parts in ten thousand, the resistance r should be of such a value that the resistance of S is not greater than 0.00025 of r. As S is seven and a half ohms, it follows that r should be at least 30,000 ohms. A resistance of 50,000 ohms was used, as that size happened to be available, and the galvanometer was of ample sensitivity. A damping shunt was affixed to the galvanometer and adjusted so that the instrument was not quite critically damped.

The resistance R is low enough in comparison with r so that the variations in the voltage of the point E, as the slider is moved from one end of S to the other (the relay being open-circuited), are negligible compared to the voltage of the battery D. If this condition is not fulfilled, the condenser will discharge to varying voltages, depending on the setting of S, and the reading of the latter will not be linear with the frequency of the discharge. A resistance of about 100 ohms was used for R, and a capacity of about 2 μf for C. Both are adjustable in small steps, although it would be sufficient if either one were so adjustable, in order to get a balance for the initial setting. The resistance W is inserted to limit the initial charging current and is not critical in value.

Values of circuit constants as used by the Bureau of Standards are as follows:

$D=90$ volts	$r=50{,}000$ ohms
$W=100$ ohms	$S=7.5$ ohms
$C=2.111$ μf	$B=2$ volts
$R=100$ ohms	

The galvanometer has a sensitivity of about 10,000 megohms, and a resistance of about 600 ohms.

IV. Theory

If the circuit conditions noted above, namely, that R and S be small compared to r, are complied with, the approximate theory

becomes very simple. Referring to Fig. 2, the moving contact of the relay F is closed in positions 1 and 2, respectively, for sufficient time so that complete charge and discharge of condenser C is obtained. The charge q on C is

$$q = Ce \tag{1}$$

where e is the voltage of battery D.

When C discharges the current takes two paths, one through R and the other through the galvanometer G. If r is so large that all the other resistances in the galvanometer circuit can be neglected, then the portion of the charge q_g which flows through the galvanometer is

$$q_g = q\frac{R}{R+r} = Ce\frac{R}{R+r}. \tag{2}$$

If there are f discharges per second then the quantity per second is

$$fq_g = fCe\frac{R}{R+r}. \tag{3}$$

The steady current through the galvanometer from the battery B assuming r large with respect to S, is

$$i_g = \frac{S_1}{S}\frac{E}{R+r} \tag{4}$$

where E is the voltage of the battery B. The adjustment is made such that the effect of the current i_g on the galvanometer is equal and opposite to the integrated effect of the pulsating current fq_g. Therefore, equating (3) and (4)

$$fCe\frac{R}{R+r} = \frac{S_1}{S}\frac{E}{R+r} \tag{5}$$

$$f = \frac{S_1}{S}\frac{E}{eCR}. \tag{6}$$

However, it is possible that the galvanometer may not integrate the pulses of current from the condenser properly, and hence may not read zero, even though (5) is satisfied. This condition is brought about by either of two causes: first, the magnetic field at the galvanometer coil may not be constant; and second, the coil itself may contain magnetic impurities.[4]

[4] For a full discussion of this effect, see Curtis and Moon, "Absolute measurement of capacitance by Maxwell's method," Bureau of Standards Scientific Paper No. 564, p. 507, ff.

An experimental method used by Curtis and Moon to determine whether or not a given galvanometer integrates correctly is illustrated by the circuit of Fig. 3. Care should be used with this arrangement in order to avoid damaging the galvanometer, especially if it is one of high sensitivity. The voltage of the battery should be small, the

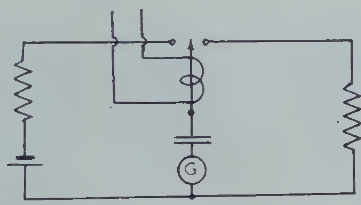

Fig. 3—Circuit arrangement used to determine suitability of galvanometer.

capacity of the condenser likewise small, and the resistances should be high enough to limit the initial currents on charge and discharge to safe values. If the galvanometer reads zero in the arrangement of Fig. 3, it may be safely assumed that it will integrate correctly in the frequency measuring network.

V. Method of Operation

A tube driven elinvar tuning fork was available for calibrating the slide-wire. The fork frequency was 99.98 cycles per sec., remaining constant at all times to within a part in ten thousand.

The method of operation is extremely simple. The tuning fork output is applied at A and the slider on S set so that the scale reading is 999_8 (the last figure being beyond the calibration, must be estimated). Then the capacity of the condenser, the resistance R, or both, are varied until the galvanometer reads zero. If, now, the alternating current, the frequency of which is to be measured, is applied at A, the slider of S is moved until the galvanometer again reads zero, all the other circuit constants remaining invariant. The frequency desired can then be read directly from the scale of the slide-wire. For instance, if the setting is 724_9, the frequency is 72.4_9 cycles per sec.

If measurements of high precision are desired, it is advisable to redetermine the setting for the standard frequency before each series of measurements. This is necessary because of changes in battery voltages, and the changes in resistance and capacity of the circuit elements due to temperature changes. The battery circuits should be closed for a sufficient length of time before measurements are made so that the batteries and resistances can come to a condition of equilibrium.

For frequencies below 5 cycles per sec. the individual impulses to the galvanometer become objectionable and the balance point cannot be accurately located. However, such low frequencies can be determined readily by merely counting the swings of the needle of the milliammeter (which is in the plate circuit of the last tube of the beat note amplifier) over a short period of time. A stop watch can be conveniently used.

In order to extend the range to higher frequencies it is only necessary to set the point for calibration at some lower point of the slide-wire and multiply the scale accordingly. For example, under the conditions given above, if instead of using 99.98 for the first setting, 49.99 is used, then 100.0 will represent 200 cycles per sec. The limitation is of course in the frequency to which the relay will respond. The limitation of the relay used is about 200 cycles per sec.

VI. Accuracy Obtainable

Although the slide-wire is only claimed to be accurate to a quarter of a division, corresponding to 0.025 cycle per sec., settings on a given frequency will usually repeat to 0.01 cycle per sec.

The linearity of the scale has been verified in two distinct ways. First, a constant speed alternator with 100-cycle and 10-cycle rotors on the same shaft, assuring that the one frequency is exactly one-tenth of the other, was available. The slide was balanced at 100 cycles per sec., and then the 10-cycle frequency was measured in the regular manner, the scale reading obtained was 10.02 ± 0.005 (estimated) cycles per sec. This error is less than the error of calibration of the slide-wire.

Second, numerous checks were also made at various odd frequencies, reading the frequency on the slide, and recording it on the oscillograph at the same time. These checks consistently agree within 0.03 cycle per sec. Hence the maximum error is well within 0.1 cycle per sec. No checks were made above 100 cycles since all measurements desired were below this value.

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A NOTE ON THE MATHEMATICAL THEORY OF THE MULTIELECTRODE TUBE*

By

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Summary—*The expression for the a-c current (rather the change in current due to applied a-c voltages) in any electrode is expanded in an ascending power series in terms of all the applied a-c voltages. It is shown that the coefficients in these series must satisfy a number of systems of linear simultaneous equations (one system for each power and frequency of the terms), and that hence to obtain the coefficients in any particular case it is merely necessary to set up these equations and solve them. The solution of the equations of course increase in complexity as the number of electrodes increases.*

The development does not make any assumptions of approximations, although in the discussion only the terms of the first and second degree are considered. It is shown, however, that however slow be the convergency of the series, the coefficients must always satisfy similar sets of algebraic equations.

INTRODUCTORY

IT HAS heretofore been customary to set up a Taylor expansion for the current (usually the plate current) in the electrode considered, and by assuming a power series for the same current and equating to the Taylor series, the coefficients of the power series were determined. An investigation of these coefficients then gave the various modes of operation of the device. In the case of multielectrode tubes, however, this procedure leads to a complexity that hides the general properties of the device, although it has been applied to the four-element tube.[1] The results given herein are intended to give a more extensive and general, though less detailed view of these properties.

NOTATION

The following symbols and definitions will be used throughout. It is to be noted that the subscripts refer to the various electrodes ($a, b, \ldots,$ n). In the case where the second of a double subscript refers to a frequency such use will be self-evident and will not lead to any confusion.

E_k = instantaneous voltage between the filament and k-th electrode
I_k = instantaneous current in the k-th electrode
E_{k0} = steady d-c value of E_k
e_k = change in E_k due to applied a-c voltage
i_k = change in I_k due to applied a-c voltage.

* Dewey decimal classification: R131. Original manuscript received by the Institute, March 18, 1930.

[1] J. G. Brainerd, "Mathematical theory of four-electrode tubes," PROC. I.R.E., **17**, 1006; June, 1929.

Whence:

$$E_k = E_{k0} + e_k$$
$$I_k = I_{k0} + i_k. \qquad (1)$$

Moreover

e_k' = a-c voltage applied to the k-th electrode circuit
z_{kr} = external impedance in the k-th circuit at the frequency r.

The following are the definitions:

$$\mu_{lm} \equiv \frac{\dfrac{\partial I_l}{\partial E_m}}{\dfrac{\partial I_l}{\partial E_l}} \qquad (2)$$

$$1/r_l \equiv \frac{\partial I_l}{\partial E_l} \qquad (3)$$

$$g_{lm} \equiv \frac{\partial I_l}{\partial E_m}. \qquad (4)$$

Hence it follows that:

$$g_{lm} = \mu_{lm}/r_l \quad \text{and} \quad 1/r_l = g_{ll} \qquad (5)$$

and also since

$$\frac{\partial g_{lm}}{\partial E_s} = \frac{\partial^2 I_l}{\partial E_m \partial E_s}$$
$$\frac{\partial g_{ls}}{\partial E_m} = \frac{\partial^2 I_l}{\partial E_s \partial E_m}$$

we have that

$$\frac{\partial g_{lm}}{\partial E_s} = \frac{\partial g_{ls}}{\partial E_m} \qquad (6)$$

The following is the well-known Taylor expansion for a function of n variables

$$f(x_1 + h_1, \cdots, x_n + h_n) = f(x_1, \cdots, x_n) + {}_j\sum_1^n h_j \frac{\partial f}{\partial x_j}$$
$$+ \frac{1}{2!} {}_j\sum_1^n {}_k\sum_1^n h_j h_k \frac{\partial^2 f}{\partial x_j \partial x_k} + \frac{1}{3!} {}_j\sum_1^n {}_k\sum_1^n {}_l\sum_1^n h_j h_k h_l \frac{\partial^3 f}{\partial x_j \partial x_k \partial x_l} + \cdots \qquad (7)$$

If now we put $x_1+h_1=E_{j0}+e_j$, then

$$f(x_1+h_1,\cdots,x_n+h_n)-f(x_1,\cdots,x_n)=i_k$$

and by substituting in (7) we have

$$\begin{aligned} i_k &= \sum_{j=1}^{n} e_j\frac{\partial f}{\partial x_j}+\frac{1}{2!}\sum_{j=1}^{n}\sum_{l=1}^{n} e_je_l\frac{\partial^2 f}{\partial E_j\partial x_l} \\ &\qquad +\frac{1}{3!}\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{m=1}^{n} e_je_le_m\frac{\partial^3 f}{\partial E_j\partial E_l\partial x_m}+\cdots \\ &= \sum_{j=1}^{n} e_j\frac{\partial I_k}{\partial E_j}+\frac{1}{2!}\sum_{j=1}^{n}\sum_{l=1}^{n} e_je_l\frac{\partial^2 I_k}{\partial E_j\partial E_l} \\ &\qquad +\frac{1}{3!}\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{m=1}^{n} e_je_le_m\frac{\partial^3 I_k}{\partial E_j\partial E_l\partial E_m}+\cdots \\ &= \sum_{j=1}^{n} e_jg_{kj}+\frac{1}{2!}\sum_{j=1}^{n}\sum_{l=1}^{n} e_je_l\frac{\partial g_{kj}}{\partial E_l} \\ &\qquad +\frac{1}{3!}\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{m=1}^{n} e_je_le_m\frac{\partial^3 g_{kj}}{\partial E_l\partial E_m}+\cdots. \end{aligned} \tag{8}$$

It is to be noted that in these expression j, l, m as here used are merely so-called summation dummies. Hence, there will be no harm in rewriting (8) with different dummy subscripts that may be more convenient. Thus

$$\begin{aligned} i_k &= \sum_{f=1}^{n} e_fg_{kf}+\frac{1}{2!}\sum_{f=1}^{n}\sum_{g=1}^{n} e_fe_g\frac{\partial g_{kf}}{\partial E_g} \\ &\qquad +\frac{1}{3!}\sum_{f=1}^{n}\sum_{g=1}^{n}\sum_{h=1}^{n} e_fe_ge_h\frac{\partial^2 g_{kf}}{\partial E_g\partial E_h}+\cdots. \end{aligned} \tag{8}$$

This is the general expression for the current in the k-th electrode circuit (due to the applied a-c voltages). It is of course understood that the derivatives entering into the expression are to be evaluated at the point (E_{j0}), i.e., at the steady state condition.

As (8) is in terms of the electrode a-c potentials, to obtain a more useful expression in terms of applied voltages, it will be necessary to consider the impedance drops in the various circuits. In this case we must write

$$e'_j = e_j + z_ji_j. \tag{9}$$

Substituting in (8),

$$i_k = {}_f\sum_1^n (e'_f - z_f i_f) g_{kf} + \frac{1}{2!}\ {}_f\sum_1^n\ {}_g\sum_1^n (e'_f - z_f i_f)(e'_g - z_g i_g)\frac{\partial g_{kf}}{\partial E_g} + \cdots. \quad (10)$$

If now we also consider the wave form of e'_j, we must replace each voltage by a corresponding series of terms, thus:

$$e'_f = {}_r\sum_1^q E'_{fr}\left[\epsilon^{j(P_{fr}t+\theta_{fr})} + \epsilon^{-j(P_{fr}t+\theta_{fr})}\right].$$

Now where a quantity is made up of a sum of two terms that are the conjugates of each other, it is customary, in view of the relationship between a number and its conjugate, to use just one of the components throughout a discussion. The result is then easily modified for the case of the other and the two results are combined to give the complete solution. On this basis we may write

$$e_f = {}_r\sum_1^q e'_{fr} - {}_r\sum_1^q z_{fr} i_f \quad (11)$$

where e'_f is of the form:

$$e'_{fr} = E_{fr}\epsilon^{j(p_{fr}t+\theta_{fr})}.$$

To obtain an expression for i_k in terms of the applied voltages, let us assume a power series with undetermined coefficients:

$$i_k = {}_f\sum_1^n\left[{}_r\sum_1^q e'_{fr}K_{fr}\right] + {}_f\sum_1^n {}_g\sum_1^n\left[{}_r\sum_1^q\ {}_s\sum_1^q e'_{fr}e'_{gs}K_{fg(r+s)}\right] + \cdots. \quad (12)$$

Substitute in (10):

$$\begin{aligned}
&{}_f\sum_1^n\left[{}_r\sum_1^q e'_{fr}K_{fr}\right] + {}_f\sum_1^n\ {}_g\sum_1^n\left[\ {}_r\sum_1^q\ {}_s\sum_1^q e'_{fr}e'_{gs}K_{fg(r+s)}\right] + \cdots \\
&= {}_f\sum_1^n\Bigg\{g_{kf}\Bigg[{}_r\sum_1^q e'_{fr} - {}_h\sum_1^n\left({}_r\sum_1^q e'_{hr}z_{fr}F_{hr}\right) \\
&- {}_h\sum_1^n\ {}_j\sum_1^n\left[{}_r\sum_1^q\ {}_s\sum_1^q e'_{hr}e'_{js}z_{f(r+s)}F_{hj(r+s)}\right] - \cdots\ \Bigg]\Bigg\}. \\
&+ \frac{1}{2!}\ {}_f\sum_1^n\ {}_g\sum_1^n\Bigg\{\frac{\partial g_{kf}}{\partial E_g}\Bigg[\Bigg\{{}_r\sum_1^q e'_{fr} - {}_h\sum_1^n\left({}_r\sum_1^q e'_{hr}z_{fr}F_{hr}\right) \\
&- {}_h\sum_1^n\ {}_j\sum_1^n\left({}_r\sum_1^q\ {}_s\sum_1^q e'_{hr}e'_{js}z_{f(r+s)}F_{hj(r+s)}\right) - \cdots\Bigg\}\Bigg\{{}_r\sum_1^q e'_{gr} \\
&- {}_h\sum_1^n\left({}_r\sum_1^q e'_{hr}z_{gr}G_{hr}\right) - {}_h\sum_1^n\ {}_i\sum_1^n\left({}_r\sum_1^q\ {}_s\sum_1^q e'_{hr}e'_{js}z_{g(r+s)}G_{hj(r+s)}\right) \\
&- \cdots\Bigg\}\Bigg]\Bigg\} + \cdots\cdots.
\end{aligned} \quad (13)$$

In (13), then, we have to individuate and equate the coefficients of like powers of e'_f (and like periodicities). Taking, for example the coefficients of e'_{mr}, we get:

$$K_{mr} = g_{mk} - f\sum_1^n g_{kf}z_{fr}F_{mr}. \tag{14}$$

Writing these equations out (there is one for each electrode current which is not *a priori* zero):

$$\left\{\begin{array}{l} (1 + g_{aa}z_{ar})A_{mr} + g_{ab}z_{br}B_{mr} + g_{ac}z_{cr}C_{mr} + \cdots = g_{am} \\ g_{ba}z_{ar}A_{mr} + (1 + g_{bb}z_{br})B_{mr} + g_{bc}z_{cr}C_{mr} + \cdots\cdots = g_{bm} \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ g_{na}z_{ar}A_{mr} + g_{nb}z_{br}B_{mr} + \cdots + (1 + g_{nn}z_{nr})N_{mr} = g_{nm}. \end{array}\right. \tag{15}$$

It is to be noted that for each component of periodicity r, there will be as many such systems of equations as there are applied voltages. Again for any applied voltage there will be as many systems as this voltage as components (of different frequencies). In other words any coefficient K_{mr} as obtained from one of these systems represents the coefficient that is to be associated with the first power of the r-th component of the voltage applied to the m-th electrode circuit, when the equation for i_k is being obtained. It is necessary to remember that any of the n equations in (15), say the j-th, results from the equation (analogous to (10)) for the current i_j. If the conditions are such that this current is *a priori* known to be zero (viz., when z_j is infinite), then (15) will consist of n-l equations and the one missing will be precisely the j-th one. This is important since in those particular cases where some of the currents are zero it is necessary to know exactly how to write down the system corresponding to (15).

As an illustration of the use of (15), let us consider a four-electrode tube in which a voltage is applied only to the control grid which, however, is supposed to draw no current. No a-c voltages are applied to the other electrodes, but their external circuits contain an impedance. Designate the control grid by A, the screen grid by B, and the plate by C. Then, since $i_a = 0$ the first equation in (15) does not exist. The system reduces to

$$\left\{\begin{array}{l} (1 + g_{bb}z_{br})B_{ar} + g_{bc}z_{cr}C_{ar} = g_{ba} \\ g_{cb}z_{br}B_{ar} + (1 + g_{cc}z_{cr})\,C_{ar} = g_{ca}. \end{array}\right. \tag{15a}$$

The solution for C_{ar} can easily be obtained and reduced to

$$C_{ar} = \frac{\mu_{ca}(r_b + z_{br}) - \mu_{cb}g_{ba}z_{br}r_b}{(r_b + z_{br})(r_c + z_{cr}) - \mu_{bc}\mu_{cb}z_{br}z_{cr}}$$

which is readily recognized[1] as the coefficient of the r-th component of the applied voltage, in the expression for the plate current. Of course if an a-c voltage had been also applied to the screen grid, there would have been another set similar to (15a):

$$\begin{cases} (1 + g_{bb}z_{br})B_{br} + g_{bc}z_{cr}C_{br} = g_{bb} \\ g_{cb}z_{br}B_{br} + (1 + g_{cc}z_{cr})C_{br} = g_{cb}. \end{cases}$$

Returning now to (13) we may proceed analogously to obtain the coefficients of the terms of the second degree and of frequency, say $r+s$. The result will be:

$$\begin{aligned} K_{lm(r+s)} = &-f\sum_{1}^{n} z_{f(r+s)}F_{lm(r+s)}g_{kf} \\ &+\frac{1}{2!}\left\{\frac{\partial g_{kl}}{\partial E_m} - f\sum_{1}^{n}\frac{\partial g_{kf}}{\partial E_m}z_{fr}F_{lr} - g\sum_{1}^{n}\frac{\partial g_{kl}}{\partial E_g}z_{gs}G_{ms}\right. \\ &\left.+f\sum_{1}^{n} g\sum_{1}^{n}\frac{\partial g_{kf}}{\partial E_g}z_{fr}z_{gs}F_{lr}G_{ms}\right\}. \end{aligned} \tag{16}$$

Writing these out, we obtain a system analogous to (15):

$$\begin{cases} [1 + g_{aa}z_{a(r+s)}]A_{lm(r+s)} + g_{ab}z_{b(r+s)}B_{lm(r+s)} + \cdots = \phi_{alm} \\ g_{ba}z_{a(r+s)}A_{lm(r+s)} + [1 + g_{bb}z_{b(r+s)}]B_{lm(r+s)} + \cdots = \phi_{blm} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ g_{na}z_{a(r+s)}A_{lm(r+s)} + \cdots + [1 + g_{nn}z_{n(r+s)}]N_{lm(r+s)} = \phi_{nlm} \end{cases} \tag{17}$$

where

$$\begin{aligned} \phi_{jlm} \equiv \frac{1}{2!}\left\{\frac{\partial g_{jl}}{\partial E_m} - f\sum_{1}^{n}\frac{\partial g_{jf}}{\partial E_m}z_{fr}F_{lr} - g\sum_{1}^{n}\frac{\partial g_{jl}}{\partial E_g}z_{gs}G_{ms}\right. \\ \left.+ f\sum_{1}^{n} g\sum_{1}^{n}\frac{\partial g_{jf}}{\partial E_g}z_{fr}z_{gs}F_{lr}G_{ms}\right\} \end{aligned} \tag{18}$$

It is to be noted that the symbol $r+s$ does not represent merely the sum of the two frequencies r and s, but in general will be either the sum or the difference. In particular if $l=m$, both a d-c and a double-frequency term will result.

If it is desired to include in the expansion a term of the third degree, the procedure is entirely analogous to the above except that in the series assumed for i_k (equation (12)), a third term must be included and hence the terms representing the impedance drops in (13) will consist of three terms instead of two.

[1] See footnote 1.

The systems represented by (15) may be termed the "amplification systems" since they involve the coefficients of the terms of first degree in the applied voltage. Similarly the coëfficients themselves may be called the "amplification coefficients." In the same way we may apply the terms "modulation systems" and "modulation coefficients" to systems such as (17) and their solutions respectively.

The solutions of the amplification and modulation systems are quite simple, at least formally, and they may be written:

$$K_{mr} = (1/\delta_r)\, j\sum_{a}^{n} \frac{\partial \delta_r}{\partial u_{jk}} g_{jm} \tag{19}$$

$$K_{lm(r+s)} = (1/\delta_{(r+s)})\, j\sum_{a}^{n} \frac{\partial \delta_{r+s}}{\partial u'_{jk}} \phi_{jlm} \tag{20}$$

where the ϕ's have already been defined (18) and

$$\delta_r \equiv \begin{vmatrix} u_{aa} & u_{ab} & \cdots & u_{an} \\ u_{ba} & u_{bb} & \cdots & u_{bn} \\ \cdot & \cdot & \cdots & \cdot \\ u_{na} & u_{nb} & \cdots & u_{nn} \end{vmatrix} \qquad \delta_{(r+s)} \equiv \begin{vmatrix} u'_{aa} & u'_{ab} & \cdots & u'_{an} \\ u'_{ba} & u'_{bb} & \cdots & u'_{bn} \\ \cdot & \cdot & \cdots & \cdot \\ u'_{na} & u'_{nb} & \cdots & u'_{nn} \end{vmatrix}$$

$$u_{jk} \equiv (\Delta_j^k + g_{jk}z_{kr}) \qquad u'_{jk} \equiv (\Delta_j^k + g_{jk}z_{k(r+s)})$$

$$\Delta_j^k \begin{cases} = 1 \text{ if } j = k \\ = 0 \text{ if } j \neq k. \end{cases}$$

Note, in these definitions, the dissymmetry of u_{jk}.

The investigation of (19) and (20) for the general conditions of amplification, oscillation, etc., will be undertaken in a future paper and constitutes the next step in this general development.

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AN EARLY NOTE ON WAVE PROPAGATION

THE following letter written by Lee de Forest to G. W. Pierce on September 13, 1912, is particularly interesting at this late date in view of those advances which have been made in the study of wave propagation during the last few years.

"Dear Professor Pierce:

"I was glad to receive your comments on my *Electrician* paper, also pleased that you did not send your letter to press for the following reasons; I think you have not considered with sufficient care what a variety of imaginable atmospheric conditions might be conceived to explain the phenomena of "fading" which I have described.

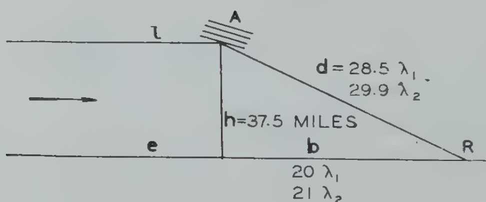

Fig. 1

"You assume only one reflecting body or stratum of air as acting, and that midway between the stations. This is surely an unnecessary restriction. As a matter of fact, it is impossible to state, within say 20–30 wavelengths, the distance between the two stations which in reality is quite immaterial in the explanation of the phenomena, so long as the distance is large compared with 10–20 wavelengths.

"Assume an upper reflecting body of air, A, at a distance of 20 in front of the receiving station, at a height h above the earth, its under surface so directed that the wave front incident thereon is reflected, or bent, down to the earth at the station R.

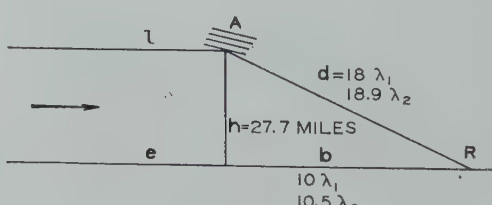

Fig. 2

"The distance from the sending station is sufficient to make l sensibly parallel with the earth's surface e. Let $\lambda_1 = 1.05\lambda_2$ where λ_1 is the wavelength of the 'sending' wave, λ_2 that of the 'compensation' wave. Now assume h such that $d = 28.5\lambda_1$. Then we shall have a perfect interference between the λ_1 waves which reach R by the earth's surface path and those which came from A, and an almost perfect reinforcement between the two λ_2 waves. Solving this triangle we get $h = 20.2\lambda_1 = 60600$ m $= 37.5$ miles—a height not at all absurd.

"Take another case, Fig. 2, where the reflecting body A is vertically above a point 30 km, in front of the receiving station ($\lambda_1 = 3000$ m). Here again the de-

sired relations between reinforcement of the two λ_1 waves and the almost complete opposition of those of the two λ_2 waves are obtained. Here $h = 15\lambda_1 = 27.7$ miles.

"These two examples of necessary values for h conform so well with what I have been taught to regard as the probable altitude of the well ionized layers, or "upper limits" of the atmosphere, as to afford a comfortable feeling of corroboration (1) although I well realize that we know almost nothing of what actually goes on up above, and that attempts at exact explanation are silly.

Suppose another arrangement of air body and receiving station, Fig. 3.

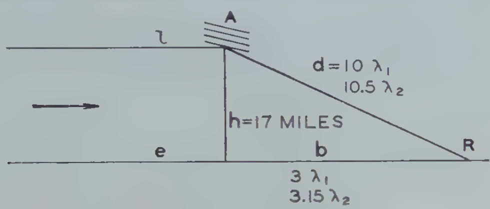

Fig. 3

"Here the two λ_1 waves arrive at R in phase while the two λ_2 waves are 126 deg. out of phase, sufficient to cause a decided diminuation in strength of signals from shorter waves. The height of A here is only 17 miles. Observe in this last example how a lifting of the reflecting body A through a height of approximately one-half a wavelength or so that d was increased a half wavelength while b remains practically the same, would produce a complete reversal of the 'fading' phenomena. Or the mere change of the angle of the deflecting surface through a few degrees would produce the same effect. This would explain very nicely the strange fadings and recoveries of our signals, sometimes requiring reversals of the sending key once in five or ten minutes.

"If we assume three reflections of the upper surface of the wave, two from ionized strata and one from the earth's surface midway between, the height necessary to explain the phenomena is reduced to one-third—say nine miles in the second example given. However, I do not think we need assume anything so fantastic. You will recognize that we have three variables, the height of the reflecting stratum, the distance of this 'mirror' from receiving station, and the angle of incidence of its surface. We can also imagine that successive reflections or deflections—and these not necessarily in a vertical plane— may occur between several masses of ionized air possessing various refractive indices, etc.

"If it is assumed that the reflection occurs at a point midway between the two stations, the minimum height necessary to cause the interference and reinforcement becomes maximum. (Incidentally you made an error in your figures, your h is 62 miles instead of 196.)

"In view of the rapidity of the changes I have described, it is quite impossible to ascribe them to any alterations in strictly '*terrestrial*' conditions. I do not see how Zenneck's work will aid us here.

"We have even observed a fading of a certain wave at one station while another receiving station *less than ten miles distant* observes nothing of the sort. At other times the same fading is observed simultaneously at both stations.

"I shall be pleased to hear from you further. Believe me,"

Very truly yours,
Lee de Forest

Palo Alto, Cal.
September 13, 1912.

"P.S. It seems significant that never, so far as I have been able to ascertain, have our operators observed a 'fading' of both λ_1 and λ_2 waves simultaneously (except of course the fading which comes at dawn, which affects all waves of the same order of wavelength alike). This fact seems to lead again to the conclusion that the reflecting bodies are at great heights, such as those we have been discussing. The nearer λ_1 and λ_2 approach each other the higher must be the reflecting body which can produce interference with one and reinforcement of the other.

"Note added.—The phenomena are absent generally except toward sevening and during the first half of the night."

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BOOK REVIEW

Radio Data Charts, by R. T. Beatty. Iliffe & Sons Ltd., Dorset House, Tudor Street, London. 82 pages. Price 4'6 net.

This is a series of charts originally published in the *Wireless World* and *Radio Review* and now revised and amplified for publication in book form. The charts have been designed to free the wireless enthusiast from the weariness of figures. Multiplication, division, and their accessories are performed by laying a ruler across two scales and reading the answer on a third. An attempt has been made to provide most of the essential data required in receiver design.

Some typical calculations which can be made with these charts are the relation of inductance and capacity to frequency, the reactance of coils and condensers at radio and audio frequencies, resistances in parallel and condensers in series, self-inductance of solenoids, diameter of wire to give coil of minimum high-frequency resistance, the design of iron-cored chokes carrying direct current, and the design of high-frequency transformers.

The text which accompanies the charts explains their construction and use and is a guide to many phases of the science and the art of radio.

S. S. Kirby*

* Bureau of Standards, Washington, D. C.

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

Copies of the publications listed on this page may be obtained gratis by addressing a request to the manufacturer or publisher.

The General Radio Co. of Cambridge, Mass., has recently issued its new catalog of instruments for electrical measurements at communication frequencies. Catalog F, a 120-page book describing the complete line of G-R products will be sent only to those who are in a position to influence the purchase of laboratory apparatus.

"Facts on Soldering" is the title of a 36-page booklet published by the Kester Solder Co., 4201 Wrightwood Ave., Chicago, giving pertinent facts on the correct use of solder.

"Resistance Thermometers" is the name of Catalog No. 80 of the Leeds and Northrup Co., 4901 Stenton Ave., Philadelphia. Controlling, recording, and indicating resistance thermometers and thermometric equipment for use in plants where quality control is essential are described.

The Rawson Electrical Instrument Co. has for distribution a loose-leaf catalog of bulletins describing single and double pivot portable and switchboard meters. Requests for bulletins or the general catalog may be addressed to the company at Cambridge, Mass., or at 91 Seventh Ave., New York. The catalog contains the following bulletins:

Bulletin 1A gives general information on the construction of single and double pivot instruments, as well as characteristics of Rawson meters.

Bulletin 501A describes the direct current multimeter, having fourteen scale ranges from 200 microamperes and 2 millivolts to 1 ampere and 1000 volts full scale deflection.

Single and multiple range direct-current portable meters are described in Bulletin 501. Either single or double pivot meters from 4 microamperes and 1.5 millivolts full scale deflection to instruments having scales from 1 ampere to 1 volt full scale deflection are available as standard products.

Bulletin 502 describes thermojunctions in vacuum, meters for use with junctions, and the AC-DC thermal multimeter. The thermal multimeter may be used on direct current or on alternating current up to 5000 cycles. Full scale deflections are available from 10 milliamperes or 0.3 volt to 3 amperes or 1000 volts.

Type 503 ultrasensitive semisuspended meter are described in Bulletin 503.

The Rawson multimeter junior multimeter, which has recently made its appearance is described in Bulletin 507A. The junior is a twelve-range direct-current instrument, the various ranges being selected by means of a rotary switch. Full scale deflections from 2 millivolts or 200 microamperes, to 1 ampere or 1000 volts are provided.

Bench and flush mounting instruments are described in Bulletin 507.

Electrostatic voltmeters are described in Bulletin 508.

The Kasson timer for timing relays is described in Bulletin 110.

A cable testing outfit is described in Bulletin 112A.

Bulletin 504 described the Rawson fluxmeter, a portable instrument for measuring lines of force, flux densities per unit area, hysterisis, and permeability of magnetic samples.

The Aerovox condenser and resistor manual on the proper use of condensers and resistors in radio receivers and power supply units also contains detailed specifications of condensers and resistors manufactured by the Aerovox Wireless Corp., 70–82 Washington St., Brooklyn, N. Y.

"Pilot Radio Products" is the name of the general catalog of tubes, complete receivers, and radio parts manufactured by the Pilot Radio and Tube Corp., of Lawrence, Mass.

REFERENCES TO CURRENT RADIO LITERATURE

THIS is a monthly list of references prepared by the Bureau of Standards and is intended to cover the more important papers of interest to the professional radio engineer which have recently appeared in periodicals, books, etc. The number at the left of each reference classifies the reference by subject, in accordance with the scheme presented in "A Decimal Classification of Radio Subjects—An Extension of the Dewey System," Bureau of Standards Circular No. 138, a copy of which may be obtained for 10 cents from the Superintendent of Documents, Government Printing Office, Washington, D. C. The various articles listed below are not obtainable from the Government. The various periodicals can be secured from their publishers and can be consulted at large public libraries.

R000. Radio Communication

R090 Eccles, W. H. Physics in relation to wireless. *Nature* (London), **125,** 894–97; June 14, 1930.

An historical account is given of the contributions of physicists to radio. The application of these contributions to the science and their importance in its development are pointed out.

R100. Radio Principles

R113.1 Krüger, K. and Plendl, H. Ueber ein Verfahren zum Ausgleich von Schwunderscheinungen bei kurzen Wellen. (Concerning a method for balancing out fading effects on short waves.) *Zeits. für Hochfrequenz.*, **35,** 191; May, 1930.

Two horizontal half-wave antennas mounted at right angles to each other are fed from separate power amplifiers which are one hundred per cent modulated and receive the carrier frequency from a common, crystal-controlled, master oscillator. With this arrangement of the transmitter marked freedom from fading effects is obtained. Variations at the receiving end are from 1 to 3 while with one antenna radiating the same power the variations are found to be from 1 to 20.

R113.4 Appleton, E. V. Some notes on wireless methods of investigating the electrical structure of the upper atmosphere. *Proc. Phys. Soc.* (London), **42,** 321–39; June 15, 1930.

The moving transmitter method of determining the height of the atmospheric ionized region is discussed. The relations between the optical and equivalent paths of waves deviated by the upper atmosphere and between the rates at which these quantities may vary with time are investigated theoretically. From the results of experiments deductions are made concerning (1) the existence of more than one ionized layer, (2) the possible influence of magnetic storms on atmospheric ionization, (3) the gradient of ionization and its alteration under solar influence at sunrise, and (4) the actual height reached by the waves deviated by the upper atmosphere.

R113.4 Hulburt, E. O. Wireless telegraphy and the ionization in the upper atmosphere. Proc. I. R. E., **18,** 1231–38; July, 1930.

The theory of ionization of the upper atmosphere by ultra-violet light from the sun and the resulting effects upon the behavior of wireless waves are surveyed. The discussion includes a treatment of the effect of the diurnal variation of the earth's magnetism on the density and drift of atmospheric ions and the resulting observed effects on skip distance and the angle of reflection of wireless waves from the Heaviside layer.

R113.6 Bureau, R. Cartes de propagation d'ondes courtes. (Propagation maps of short waves.) (Continued from p. 114, March issue.) *L'Onde Electrique*, **9,** 166–174; April, 1930.

Additional propagation maps are presented. Variations in the maps from one day to another are pointed out and possible relations between the atmosphere and the phenomena of propagation are discussed. The use of the maps in the indirect exploration of the Kennelly-Heaviside layer is considered.

R113.6 Burnett, D. The reflexion of long electromagnetic waves from the upper atmosphere. *Phil. Mag.* (London), **10**, 1–15; July, 1930.

The problem of the reflection of electromagnetic waves from the upper atmosphere is treated mathematically for the case of very long waves. The treatment is supplementary to one by MacDonald which is valid for shorter waves. The amplitude of the magnetic force in the difracted wave plus a wave reflected once from the upper atmosphere, expressed as a fraction of the amplitude at the same point due to the same transmitter in free space, is tabulated for various angular distances.

R113.7 Eckersley, P. P. The calculation of the service area of broadcast stations. Proc. I. R. E., **18**, 1160–93; July, 1930.

The practical considerations in the determination of the field strength laid down at any distance by a given radio station at any given location are comprehensively studied. All factors entering into such calculations are considered and practical means of their determination or estimation set forth. Methods are provided for the evaluation of field strength due to the space ray which should prove of value in further work on this subject. No attempt at extreme accuracy is made, the keynote of the study being practicability.

R113.7 Ratcliffe, J. A. and White, F. W. G. Negative attenuation of wireless waves. *Nature* (London), **125**, 926–27; June 21, 1930.

A study of the attenuation of the signal of the Daventry station of the British Broadcasting Co., indicated a negative attenuation within a certain distance of the transmitter. A proposed investigation of this phenomenon is outlined. Rolf's numerical calculations on Sommerfeld's attenuation theory are used as the basis of a possible explanation.

R114 Barkhausen, H. Whistling tones from the earth. Proc. I. R. E., **18**, 1155–59; July, 1930.

Two possible explanations of the phenomenon known as whistling tones, described by the author in a communication in 1919, are presented. Both assume an electromagnetic impulse caused by a stroke of lightning In the first, the tone is the impulse and its reflections from the Heaviside layer arriving at the receiver-amplifier with such time intervals as to give an audio note. In the second, the tone is the audio frequencies of the impulse arriving in a certain sequence because of the differences in propagation velocities of these frequencies.

R132 Forstmann, A. Ueber die Erzielung unverzerrter Maximalleistungen durch Endverstärkerröhren bei Anwendung nichtlinearer Schwingungen. (On obtaining maximum undistorted power output from the last audio amplifier stage with tubes operating on nonlinear characteristics.) *Elek. Nach. Technik*, **7**, 203–10; May, 1930.

The problem of obtaining maximum power output with optimum efficiency and minimum distortion from terminal amplifier tubes operating on the non-linear parts of the characteristics is considered. The consideration covers the cases of single and two grid tubes.

R134 Turner, P. K. Grid or anode rectification? *Experimental Wireless & W. Engr.* (London), **7**, 371–75; July, 1930.

The superiority of grid rectification over plate rectification in receiving sets designed for broadcast reproduction is pointed out. The distortion introduced by each system under varying conditions is presented graphically and the two are compared. The two methods of rectification are also compared with respect to frequency response, problems of coupling, flexibility, and efficiency.

R135 Godfrin, P. Note sure la stabilité de l'accrochage. (Note on the stability of tuning.) *L'Onde Electrique*, **9**, 190–96; April, 1930.

The properties of a regenerative receiver employing grid detection nearly tuned to an oscillation are studied. By a single illustrative case it is shown that under certain circumstances no stable region of oscillation can exist. This is presented as an explanation of the low frequency parasitic noise commonly known as threshold howl.

R138 Lowry, E. F. The role of the core metal in oxide-coated filaments. *Phys. Rev.* **35**, 1367–78; June 1, 1930.

The enormous difference found between the emission characteristics of two types of oxide-coated filament necessitates the conclusion that the core metal has a definite function other than simply a mechanical support. In order to account for the difference a modification is suggested in existing ideas concerning the mechanism of emission from

this type of cathode. The source of emission is assumed to be the composite layer formed by the occlusion of the alkaline earth metal on the surface of the core, the electrons emitted being diffused through the interstices in the oxide-coating into the vacuous space. It is shown that this explanation accounts for other peculiarities in the behavior of oxide cathodes.

R140 Reed, M. Electrical wave filters. (Continued from p. 322, June issue.) *Experimental Wireless & W. Engr.* (London), **7**, 382–86; July, 1930.

As an introduction to a consideration of the subject of composite wave filters the general formulas for the image impedances and the transfer constant are derived, first, of a single unsymmetrical structure and, then, of a combination of two or more unsymmetrical structures in series. The formulas for the image impedance and transfer constant of a half section are also found (to be concluded).

R140 Watanabe, Y. Betriebsdiagramme für symmetrische Kettenleiter. (Working diagrams for symmetrical filters.) *Elek. Nach. Technik*, **7**, 153–66; April, 1930.

The mathematical expressions are developed for the impedance and propagation constant of several types of filter. These include the low-pass, high-pass, band-pass, and band-suppression types of the usual design. Graphs showing the impedance frequency characteristics of the various types are included.

R141.2 Bedeau, F. and DeMare, J. Application de la théorie de l'inversion à la construction de courbes de résonance. (Application of the theory of inversion to the construction of resonance curves.) *L'Onde Electrique*, **9**, 178–89; April, 1930.

A method is demonstrated of obtaining by a single construction not only the resonance curve of a circuit having a predetermined coefficient of overvoltage ($=L\omega_0/R$) but also those corresponding to circuits of which the coefficients of overvoltage are larger or smaller in geometric progression. It is shown that the method applies also to antiresonant circuits and that the resonance curves of the resonant circuits and the impedance curves of the antiresonant circuits are the same.

R145 Barclay, W. A. Applications of the method of alignment to reactance computations and simple filter theory. Part V. (Continued from p. 314, June issue.) *Experimental Wireless & W. Engr.* (London), **7**, 376–81; July, 1930.

Alignment charts for the high-pass, low-pass, and band-pass symmetrical T filters when used with a resistance load are presented. These permit the ready determination of the response curves of these filters. Sample response curves obtained by the use of the charts are reproduced. Certain modifications of the band-pass symmetrical T filter are discussed and an alignment chart is given which is designed for the determination of the response curve of the modified filter.

R148 van der Pol, B. Frequency modulation. Proc. I. R. E., **18**, 1194–1205; July, 1930.

The differential equation of a frequency modulated transmitter is considered and the expression for the current as a function of time is derived. A frequency analysis of this function is made for two specific cases: (1) sinusoidal frequency modulation (telephony) and (2) right-angle frequency modulation (telegraphy). The distribution and amplitudes of the frequencies present in each case are discussed. Charts are included showing the derived frequency spectra.

R162 Beatty, R. T. The numerical expression of selectivity. *Experimental Wireless & W. Engr.* (London), **7**, 361–366; July, 1930.

A numerical definition of selectivity is proposed and a simple formula is derived which allows this quantity to be easily calculated for any high frequency amplifier which does not employ regeneration or high-frequency filter circuits.

R200. Radio Measurements and Standardization

R201.7 McNeely, J. K. and Konkle, P. J. Locating radio interference with the oscillograph. Proc. I. R. E., **18**, 1216–1225; July, 1930.

An oscillographic method of locating radio interference is described. It is shown that by careful comparisons of field oscillograms with oscillograms of disturbances from known sources accurate predictions of the sources of the interference can be quickly made.

R214 Giebe, E. and Scheibe, A. Transversalschwingende Leuchtresonatoren als Frequenznormale im Bereich von 1000–20,000 Hertz. (Light resonators vibrating transversely as frequency standards in the range 1–20 kc.) *Zeits. für Hochfrequenz.*, **35**, 165–177; May, 1930.

The behavior of quartz plates in vacuum as frequency indicators is theoretically considered. The problems involved in using the resonance illumination property of quartz in the design of a frequency standard are discussed. Such features as auxiliary circuits and apparatus, and effect of temperature upon frequency are treated.

R214 Heaton, V. E. and Brattain, W. H. Design of a portable temperature-controlled piezo oscillator. Proc. I. R. E., **18**, 1239–46; July, 1930.

The essential details of a portable shielded temperature-controlled piezo oscillator, constant in frequency to better than 1 part in 100,000, are described. The circuit arrangement, the temperature control, and the quartz plate mounting are explained.

R214 Mögel, H. Einige Methoden zur Frequenzmessung von kurzen Wellen. (A method for frequency measurement of short waves.) *Elek. Nach. Technik*, **7**, 133–140; April, 1930.

An exposition is given of the application of quartz glow tubes as frequency standards. A discussion of the allied circuits needed to make the frequencies available throughout the laboratory is included.

R242 Moullin, E. B. Notes on the detuning method of measuring the high-frequency resistance of a circuit. *Experimental Wireless & W. Engr.* (London), **7**, 367–70; July, 1930.

The reactance variation method of measuring the high-frequency resistance of a circuit is reviewed. Sources of error in the method are pointed out and special attention is given to the error due to small changes of frequency in the radio-frequency source.

R260 Stuart, W. S. A thermionic valve potentiometer for audio frequencies. *Jour. I. E. E.* (London), **68**, 769–772; June, 1930.

A description is given of a method of measuring audio frequency potential vectors with the aid of thermionic tubes. The arrangement used is adapted for tests on circuits such as artificial lines. A method of estimating the admittance existing between the input potential terminals is described.

R269 Macalpine, W. W. A radio-frequency potentiometer. Proc. I. R. E., **18**, 1144–54; July, 1930.

A potentiometer developed for the measurement of the amplitude and phase of voltages at frequencies up to at least 10^6 cycles per second is described. Two slide wires are used which carry currents whose phases are approximately in quadrature. A method of standardizing the instrument is given by which the ratio of the currents in the two slide wires or dials may be determined to within 1 percent, and their phases to 1/2 degree.

R300. Radio Apparatus and Equipment

R381 Ramsay, R. R. The variation of the resistance of a radio condenser with capacity and frequency. Proc. I. R. E., **18**, 1226–1230; July, 1930.

Methods of measuring the resistance of a radio condenser are briefly reviewed and discussed. An empirical formula is presented to reduce the resistance of a condenser to its resistance at standard conditions ($\lambda = 300$ m., $C = .001$ mfd). Its use permits the comparison of condensers. Tables are reproduced giving the results of measurements of the resistance of variable radio condensers by various investigators.

R400. Radio Communication Systems

R402 Brown, W. J. Ultra-short waves for limited range communication. Proc. I. R. E., **18**, 1129–43; July, 1930.

Experiments on short distance communication using a wavelength of 2 meters, together with details of equipment used, are described. A range of over twelve miles was obtained with a super-regenerative receiver. The advantages of a limited range short wave system for certain maritime and other purposes are listed.

R412 Le Corbeiller, Ph. and Valensi, G. Notions générales de transmission appliquées à la radiotéléphonie. (General notions of transmission applied to radiotelephony.) *L'Onde Electrique*, **9**, 141–165; April, 1930.

The principal features of the transmission of a sinusoidal sound over an electrical telephone circuit are examined. Voltage level diagrams are illustrated by the diagrams of an actual land cable circuit and of the London-New York radiotelephone circuit. The principal results of speech sound analysis made recently in the United States are given and an exposition is made of the present technique employed in acoustic tests of telephone apparatus.

R500. Applications of Radio

R510 David, P. Les procédés radio-électriques pour le guidage des navires et des aéronefs. (Radio-electric processes for guiding ships and aircraft.) *L'Onde Electrique*, **9**, 197–228; May, 1930.

A review is presented of the various systems in which radio is applied as an aid to maritime and aeronautic navigation. The advantages, disadvantages, and practical value of the systems rather than their technical details are described. The systems are separated for discussion into those which give the moving craft its location with respect to a straight line, either fixed, or variable and known, and those which permit the craft to trace a curved course.

R800. Nonradio Subjects

537.65 Cady, W. G. Electrostatic and pyro-electric phenomena. Proc. I. R. E., **18**, 1247–62; July, 1930.

The more important available data gathered from many sources in the fields of electrostriction, piezo-electricity, and pyro-electricity are summarized. References are given to sources where further information may be obtained.

537.87 Malov, N. N. and Rschevkin, S. N. Widerstand des menschlichen Körpers bei hochfrequenten elektrischen Strömen. (Resistance of the human body to high-frequency electric currents.) *Zeits. für Hochfrequenz.* **35**, 177–191; May, 1930.

The methods employed in determining the resistance characteristics of human tissue are discussed. The dependence of the high-frequency resistance and capacity of the human body on the method of measurement is pointed out. The electric energy absorption of the human body is shown to be similar to that of a common salt solution.

621.313.7 Pelabon, H. Sur le mécanisme de la rectification dans le redresseur à oxydule de cuivre. (On the mechanism of rectification in the copper-oxide rectifier.) *L'Onde Electrique*, **9**, 229–244; May, 1930.

It is shown that oxidized copper owes its rectifying power to the existence of an interior bad contact which is formed during the oxidation of the metal in air. The direction of the rectified current experimentally obtained supports this theory whereas the direction is theoretically wrong if one considers as the only bad contact that formed by the one electrode placed on the oxidized face of the other. The influences of pressure, voltage, and the nature of the contact electrode are studied.

621.385.95 Ballantine, S. Effect of cavity resonance on the frequency response characteristic of the condenser-microphone. Proc. I. R. E., **18**, 1206–1215; July, 1930.

The irregularities in the response of a condenser microphone are considered from a theoretical point of view, and the results thus obtained are checked by sound pressure measurements with a Rayleigh disc. A design is given for a microphone modified to eliminate the cavity before the membrane.

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